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EPTD NOTE
BEC ,BBSR

Types of conductors :-

Transmission line conductors are usually aluminium. It is because aluminium conductors have much lower cost and lighter weight compared to other conductors like copper. An aluminium conductor has a larger diameter than a Cu conductor of same resistance.

As aluminium conductor has a large diameter, hence the lines of electric flux originating on the conductor will be farther apart at the surface for the same voltage. This means there is a lower potential gradient at the conductor surface and less tendency to ionize the air around the conductor.

These are the following aluminium conductors are as follows :

(i)	AAC	all aluminium conductors.
(ii)	AAAC	all aluminium alloy conductors.
(iii)	ACSR	aluminium conductor steel reinforced.
(iv)	ACAR	aluminium conductor, alloy reinforced.

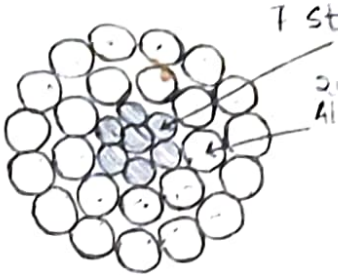
Aluminium-alloy conductors have higher tensile strength than other conductors.

ACSR consists of a central core of steel strands surrounded by layers of aluminium strands.

Figure shows the Cross-sectional view of ACSR Cable.

n Conductor shown has steel strands forming central core, around vich there are two layers aluminium strands.

→ re are 24 aluminium trands in two outer layers.



A type of conductor known as expanded Ac in which paper separates the inner steel strands from the outer aluminium stran The paper gives a larger diameter for given conductivity and tensile strength. Expanded ACSR is used for extra-high-vol lines (EHV).

Resistance:-

→ The opposition offed by a conductor to the flow of current is called as resistance (R).

→ The resistance of transmission line Conductors is the most important cause of power loss (I^2R) in a transmission l

→ The resistance of a Conductor $R = \rho \frac{l}{A}$ ⟶ (1)

Where ρ = resistivity of the Conductor
l = Length of the Conductor.
A = area of cross-section of the Conducto

→ The resistivity of the conductor depends on material of conductor and its temp.

If ρ_1 and ρ_2 are two values of resistivity at two different temp. (t_1 & t_2) then,

$$\rho_2 = \rho_1 \left[1 + \alpha(t_2 - t_1)\right]$$

Where α = temp. coeff. of resistance of the material.
This α depends on the initial temp.

Let α_0 = temp. coeff of resistance at $0°C$
α_t = " " " " " $t°C$.

$$\therefore \alpha_t = \frac{\alpha_0}{1 + \alpha_0 t}$$

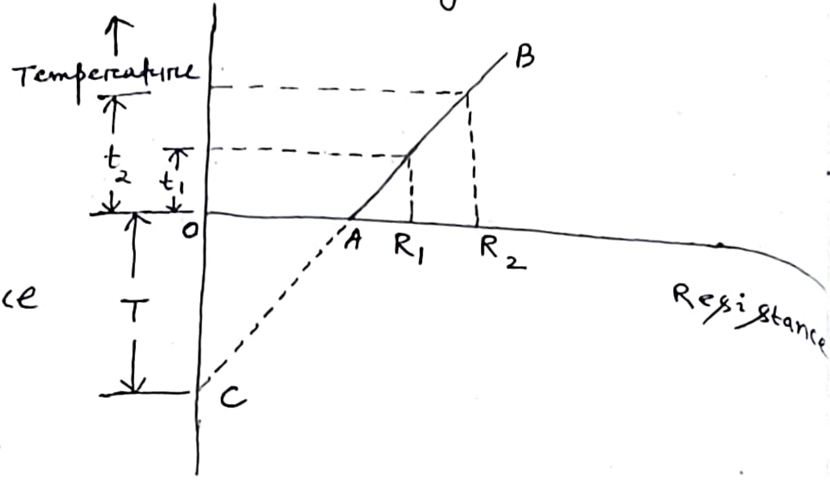
→ The resistance of stranded conductors is greater than the value obtained by Equation (1). It is because the spiraling of the strands makes them longer than the conductor itself. The increased resistance due to spiraling is calculated as 1% for three-strand conductors and 2% for concentrically stranded conductors.

→ The Resistance of a conductor also depends on its temperature. If temp. increases then resistance of the conductor increases.

If we plot a graph between resistance and temp. taking resistance as horizontal axis and temperature as vertical axis then the graph is straight line(AB) as shown in the figure.

If the graph is produced back then it intersects the temperature axis at point 'C'.

At point 'C' the resistance of the conductor is zero.



OC = T = a constant of the material of the conductor.

In simple geometry, $$\frac{R_2}{R_1} = \frac{T+t_2}{T+t_1}$$

Where R_1 and R_2 are the resistances of the conduct at temperatures t_1 and t_2 respectively in °C and 'T' is the constant determined from the graph.

For annealed cu of 100% conductivity T = 234.5 °C

For hard-drawn cu of 97% conductivity T = 241 °C

For hard drawn Al of 61% conductivity T = 228 °C

1 Electrical characteristics of aluminium Marigold stranded conductors list a dc resistance of 0.01558 Ω per 1000 ft at 20°C and ac resistance of 0.0956 Ω/mile at 50°C. The conductor has 61 strands and its size is 1113000 circular mils (cmil). ~~Vering~~ Verify the dc resistance and find the ratio of ac to dc resistance.

tion:- DC resistance, $R_0 = \rho \frac{l}{A} \longrightarrow (1)$

The dc resistance of stranded conductors is greater than the value computed by equation ① being spiraling of the strands makes them longer than the conductor itself.

Let the increased resistance due to spiraling is 2%

t 20°C from eqn ① with an increase of 2% of spiraling,

$$R_{20} = \rho \frac{L}{A}\left(1+\frac{2}{100}\right) =$$

$$= \frac{17 \times 1000}{1113000}(1.02) \quad \left[\rho = 17 \ \frac{\Omega \cdot \text{cmil}}{\text{ft}}\right]$$

$$= 0.01558 \ \Omega \text{ per } 1000 \text{ ft.}$$

Given ac resistance $R = 0.0956 \ \Omega/\text{mile} = \frac{0.0956}{5.280} \ \frac{\Omega}{1000 \text{ft}}$

50°C the resistance R_{50}

$$\therefore \frac{R_{50}}{R_{20}} = \frac{T+t_1}{T+t_2} = \frac{228+50}{228+20} \quad \left[\because T = 228 \text{ for Al}\right]$$

$$\Rightarrow \frac{R_{50}}{R_{20}} = 1.12096 \quad \Rightarrow R_{50} = R_{20} \times 1.12096 = 0.01558 \times 1.12096$$

$$\cancel{\Rightarrow R_a = \frac{R}{1.12096}} \quad \Rightarrow R_{50} = 0.0174 \ \Omega \text{ per } 1000 \text{ ft.}$$

At 50°C a.c resistance is $R = \frac{0.0956}{5.28} \frac{\Omega}{1000\ ft}$

$$\therefore \frac{R}{R_{50}} = \frac{0.0956}{5.28} \Big/ 0.01746$$

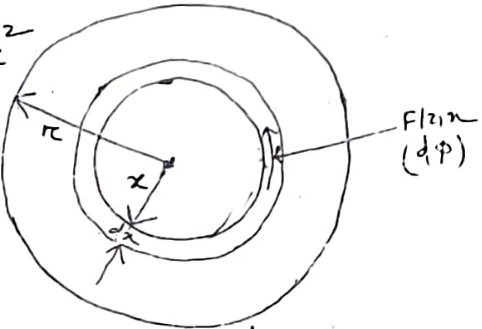
$$\Rightarrow \frac{R}{R_{50}} = 1.037$$

skin effect causes 3.7% $\left(\text{i.e } \frac{1.037 - 1}{1} \times 100 = 3.7\%\right)$ increase in resistance.

nductance of a conductor due to internal flux:

hen current flows through a conductor then magnetic field is created surrounding it. e magnetic lines of force will exist nside the conductor as well as outside e conductor. Both of these fluxes produce nductance of the conductor. Here we first onsider the internal flux and calculate the inductance.

Let us consider a long straight cylindrical conductor of radius 'r' and carrying current 'I' as shown in figure. The area of cross-section of the conductor $= \pi r^2$



Consider an element of thickness dx and radius x.

I_x = Current of the conductor of radius 'x'.

I = Current of the conductor of radius 'r'.

ssuming Uniform current density,

$$\frac{I_x}{\pi x^2} = \frac{I}{\pi r^2}$$

$$\Rightarrow I_x = \frac{\pi x^2}{\pi r^2} I = \frac{x^2}{r^2} I$$

Current density (J) $= \frac{I}{A}$

$= \frac{\text{Current}}{\text{Area}} = A/m^2$

$$L = \frac{N\phi}{I}$$

ux density x meters from the center of the conductor is,

$$B_x = \frac{\mu I_x}{2\pi x} = \frac{\mu}{2\pi x} \times \frac{x^2}{r^2} I = \frac{\mu x I}{2\pi r^2}$$

e flux through the element of thickness 'dx' and axial length one metre is,

flux = flux density × Area

$\Rightarrow d\phi = B_x \times 1 \times dx$ $\quad$ ($\because$ Area $= 1 \times dx$)

Flux per meter of length is,

$$\Rightarrow d\phi = \frac{\mu x I}{2\pi r^2} \times dx \;, \frac{wb}{meter}$$

$\therefore$ Flux Linkages through the element is given by,

$$d\lambda = \text{Flux} \times \text{No of turns}$$

$$\Rightarrow d\lambda = d\phi \times \frac{x^2}{r^2}$$

$$\Rightarrow d\lambda = \frac{\mu x I}{2\pi r^2} \times \frac{x^2}{r^2} \times dx$$

$$\Rightarrow d\lambda = \frac{\mu I x^3}{2\pi r^4} \times dx \; \frac{wb-turns}{meter} .$$

Total flux linkages from centre of the conductor to its surface is given by.

$$\int_0^{\lambda} d\lambda = \int_0^{r} \frac{\mu I x^3}{2\pi r^4} \cdot dx \quad \frac{wb-turns}{meter}$$

$$\Rightarrow \lambda = \frac{\mu I}{8\pi} \quad \frac{wb-turns}{meter .}$$

Note :-

Let N_1 = no of turn of Cond of radius r ca current I.

N_2 = no of turns of conducto of radius x carryin current I_x.

In N_1 turn the current is

In 1 " " " "

In N_2 " " " " $\frac{I}{N_1}$

But in N_2 turn the current =

$$\therefore I_x = \frac{I}{N_1} \cdot N_2$$

$$\Rightarrow N_2 = \frac{I_x}{I} N_1 = \frac{\frac{x^2 I}{r^2}}{I} N$$

$$\Rightarrow N_2 = \frac{x^2}{r^2} N_1$$

$$\Rightarrow N_2 = \frac{x^2}{r^2} \quad \left\{ \therefore N_1 = 1 \right\}$$

$$= \frac{4\pi \times 10^{-7} \, I}{8\pi} = \frac{1}{2} I \times 10^{-7} \quad \frac{wb-turns}{meter} .$$

$$\left\{ \begin{array}{l} \mu = \mu_0 = 4\pi \times 10^{-7} \\ \quad \text{air} \\ \mu_r = 1 \end{array} \right.$$

$\therefore$ Inductance per meter length of a conductor is

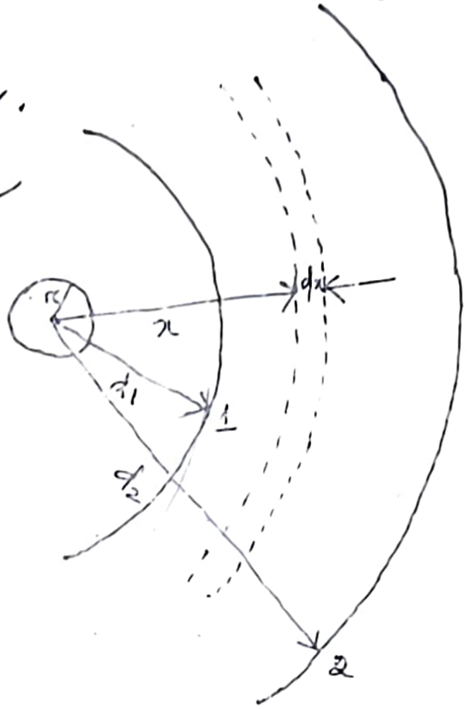
$$L = \frac{\text{flux linkages}}{\text{Current}} = \frac{\lambda}{I} = \frac{\frac{1}{2} I \times 10^{-7}}{I}$$

$$\Rightarrow L = \frac{1}{2} \times 10^{-7} \; \frac{Henrys}{mets}$$

Inductance of a Conductor due to External Flux :-

When current flows through a conductor then magnetic lines of force (i.e flux) exist outside the conductor as shown in figure.

In figure 'r' is the radius of the conductor. A magnetic field is produced surrounding the conductor whose magnetic lines of force are concentric circles as shown.



Consider two points 1 and 2 at distances d_1 and d_2 from center of the conductor.

Let an element of thickness 'dx' at a distance 'x' from center of the conductor is consider.

Flux density at distance 'x' meters from center of the conductor is, $B_x = \frac{\mu I}{2\pi x}$ wb

The flux through the element of thickness 'dx' and axial length one meter is,

Flux = flux density × Area

$\Rightarrow d\phi = B_x \times 1 \times dx$ wb

$\therefore$ Flux per meter length is, $d\phi = B_x \times dx \ \frac{wb}{m}$

$$d\phi = \frac{\mu I}{2\pi x} \cdot dx \quad \frac{Wb}{m}$$

∴ Flux linkages through the element is given by.

$$d\lambda = \text{Flux} \times \text{no of turns},$$

$$= d\phi \times 1$$

$$= d\phi$$

$$= \frac{\mu I}{2\pi x} \cdot dx \quad \frac{\text{Wb-turns}}{\text{meter}}.$$

Note
Here no of turns of
conductor of radius
is $N_1 = 1$ which carr
current 'I'.

Total flux linkages between points 1 and 2 -

$$\int_0^{\lambda} d\lambda = \int_{d_1}^{d_2} \frac{\mu I}{2\pi x} \cdot dx$$

$$\Rightarrow \lambda = \frac{\mu I}{2\pi} \log_e \frac{d_2}{d_1} = \frac{4\pi \times 10^{-7} I}{2\pi} \log_e \frac{d_1}{d_1}$$

[$\because \mu =$
$=$
For a

$$\Rightarrow \lambda = 2 \times 10^{-7} I \log_e \frac{d_2}{d_1}$$

∴ The inductance due to external flux beteen points 1 and 2 is,

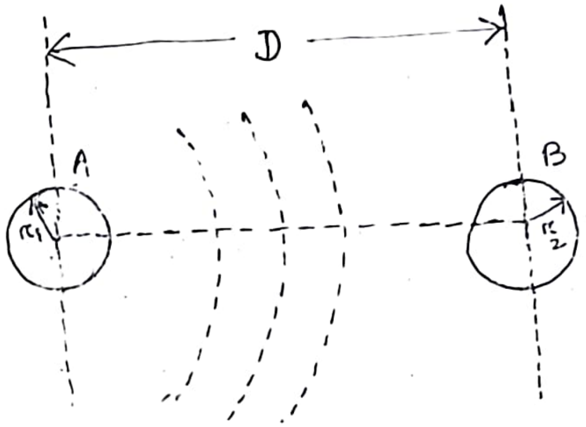
$$L = \frac{\text{Flux linkage}}{\text{Current}} = \frac{\lambda}{I} = \frac{2 \times 10^{-7} I \log_e \frac{d_2}{d_1}}{I}$$

$$\Rightarrow L = 2 \times 10^{-7} \log_e \frac{d_2}{d_1}. \quad \frac{\text{Henrys}}{\text{meter}}.$$

Inductance of a Single Phase two-wire Line :-

→ Let us consider a single phase line consists of two parallel conductors A and B having radius r_1 and r_2 separated by D meter apart. One conductor is the return circuit for the other. The two conductors carry two currents which are equal in magnitude and opposite in directions (i.e 'I' in conductor 'A' & $-I$ in B).

→ Consider only the flux linkages of the circuit due to current in conductor 'A'.



→ A line of flux produced due to current in conductor 'A' at a distance equal to or greater than $D+r_2$ from center of conductor 'A' does not link the circuit.

→ According to right hand thumb rule, the magnetic lines of force around conductor 'A' is clockwise and in conductor 'B' is anticlockwise. So two magnetic fields are in same direction at distance $D-r_2$ to centre of conductor 'B' and opposite in direction beyond distance D. As a result all the flux produced by current in conductor 'A' links all the current upto the centre of conductor 'B' and the flux beyond the centre of conductor 'B' does not link any current.

→ Flux linkages of conductor A due to external flux is given by, $\tau_{ext} = 2\times10^{-7}\, I \log_e \frac{D}{r_1}$ $\frac{\text{wb-turns}}{\text{metre}}$

→ Flux linkages of conductor 'A' due to internal flux is given by,

$$\lambda_{int} = \frac{1}{2} I \times 10^{-7} \ \frac{\text{wb-turns}}{\text{metre}}$$

→ ∴ Total flux linkages of conductor 'A' is,

$$\lambda_A = \lambda_{ext} + \lambda_{int} = 2\times10^{-7} I \log_e \frac{D}{r_1} + \frac{1}{2} I \times 10^{-7}$$

$$= I \times 10^{-7} \left(2 \log_e \frac{D}{r_1} + 0.5 \right) \ \frac{\text{wb-turns}}{\text{metre}}$$

→ Inductance of conductor 'A' is,

$$L_A = \frac{\lambda_A}{I} = \frac{I \times 10^{-7} \left(2 \log_e \frac{D}{r_1} + 0.5 \right)}{I}$$

$$= 10^{-7} \left(2 \log_e \frac{D}{r_1} + 0.5 \right)$$

$$= 2\times10^{-7} \left(\log_e \frac{D}{r_1} + 0.25 \right)$$

$$= 2\times10^{-7} \left(\log_e \frac{D}{r_1} + \log_e e^{1/4} \right) \quad \left\{ \because \log_e e^{1/4} = 0.2 \right.$$

$$= 2\times10^{-7} \log_e \frac{D e^{1/4}}{r_1}$$

$$= 2\times10^{-7} \log_e \frac{D}{r_1 e^{-1/4}} \ \frac{\text{Henerys}}{\text{metre}}$$

$$= 2\times10^{-7} \log_e \frac{D}{r_1'} \ \frac{H}{m} \quad \left\{ \because r_1' = r_1 e^{-1/4} = 0.7788 r_1 \right.$$

→ Similarly inductance of conductor B is,

$$L_B = 2\times10^{-7} \log_e \frac{D}{r_2'} \ H/M \qquad r_2' = r_2 e^{-\frac{1}{4}} = 0.7788 r_2$$

→ ∴ Loop inductance of the line, $L = L_A + L_B = 2\times10^{-7}\left(\log_e \frac{D}{r_1'} + \log\right.$

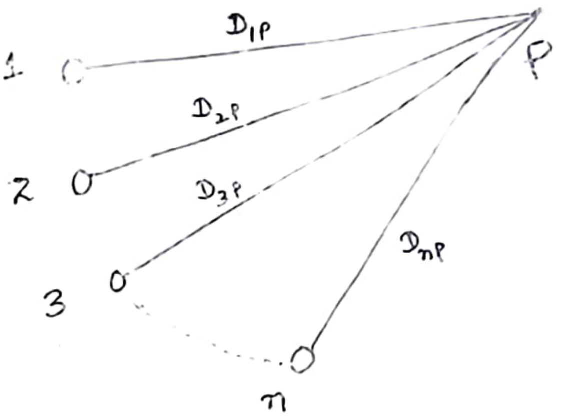
If $r_1' = r_2' = r'$, then $L = 4\times10^{-7} \log_e \frac{D}{r'} \ \frac{H}{M.}$

Flux linkages of one conductor in a group of conductors:-

Let us consider a group of parallel conductors 1, 2, 3 ---- n carrying currents $I_1, I_2, I_3 \cdots I_n$ respectively as shown in fig. The sum of the currents in various conductors is zero, i.e

$$I_1 + I_2 + I_3 + \cdots + I_n = 0$$

Let us assume that the flux linkages extend upto a point 'p' called as remote point.



Let D_{1P}, D_{2P}, D_{3P} ----- D_{nP} are the distances of the conductor from point 'p'.

The current in each conductor sets up a certain flux due to its own current. The total flux linkages of any one conductor is the sum of its linkages with all the individual fluxes set up by the conductors of the system.

Let us first calculate the flux linkages of conductor 1 due to its own current I_1 and flux linkages of conductor 1 due to the currents of the other conductors.

Flux linkages of conductor 1 due to its own current

$$\lambda_{1P_1} = I_1 \times 10^{-7} \left(2 \log_e \frac{D_{1P}}{r_1} + 0.5\right)$$

$$= 2\times 10^{-7} I_1 \left(\log_e \frac{D_{1P}}{r_1} + 0.25\right)$$

$$= 2\times 10^{-7} I_1 \left(\log_e \frac{D_{1P}}{r_1} + \log_e e^{1/4}\right) \quad \left[\because \log_e e^{1/4} = 0.25\right.$$

$$= 2\times 10^{-7} I_1 \log_e \frac{D_{1P}}{r_1 e^{-1/4}}$$

$$= 2\times 10^{-7} I_1 \log_e \frac{D_{1P}}{r_1'} \qquad \frac{\text{wb-turns}}{\text{metre}}$$

Flux linkages of conductor '1' due to current in conductor '2' (i.e I_2) within limiting distances D_{2P} and D_{12} from conductor '2' is,

$$\lambda_{1P_2} = 2\times 10^{-7} I_2 \log_e \frac{D_{2P}}{D_{12}}$$

Similarly $\lambda_{1P_3} = 2\times 10^{-7} I_3 \log_e \frac{D_{3P}}{D_{13}}$

and $\lambda_{1P_n} = 2\times 10^{-7} I_n \log_e \frac{D_{nP}}{D_{1n}}$.

Total flux linkages of conductor 1 due to currents in all conductors,

$$\lambda_{1P} = \lambda_{1P_1} + \lambda_{1P_2} + \lambda_{1P_3} + \cdots + \lambda_{1P_n}$$

$$= 2\times 10^{-7} \left[I_1 \log_e \frac{D_{1P}}{r_1'} + I_2 \log_e \frac{D_{2P}}{D_{12}} + I_3 \log_e \frac{D_{3P}}{D_{13}} + \cdots + I_n \log\right.$$

$$= 2\times 10^{-7} \left(I_1 \log_e \frac{1}{r_1'} + I_2 \log_e \frac{1}{D_{12}} + I_3 \log_e \frac{1}{D_{13}} + \cdots + I_n \log_e \frac{1}{D_{1n}}\right.$$

$$+ 2\times 10^{-7} \left(I_1 \log_e D_{1P} + I_2 \log_e D_{2P} + I_3 \log_e D_{3P} + \cdots + I_n \log_e D_{nP}\right.$$

To account for the total flux linkages of conductor 1, the point 'P' moves to infinity.

∴ $D_{1P} = D_{2P} = D_{3P} = \cdots = D_{nP} = D$ (say).

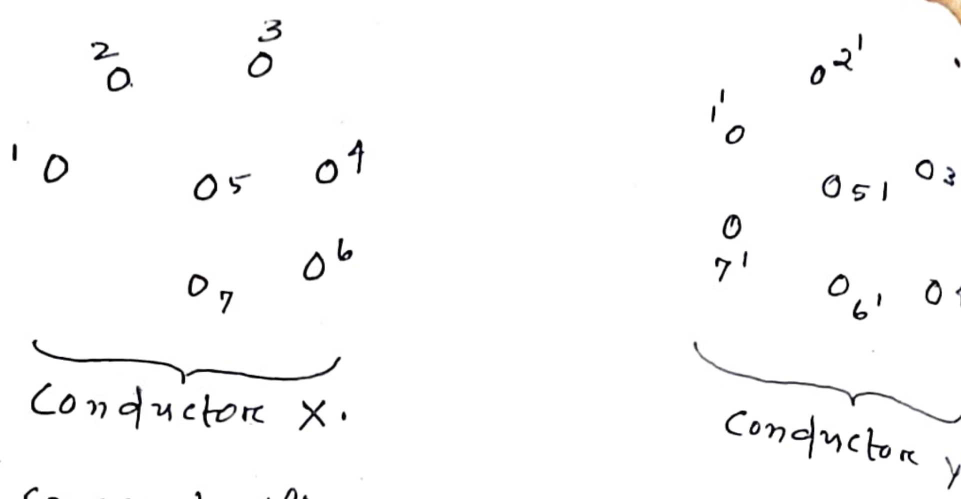
$$\lambda_{1P} = 2\times10^{-7}\left(I_1 \log_e \frac{1}{r_1'} + I_2 \log_e \frac{1}{D_{12}} + I_3 \log_e \frac{1}{D_{13}} + \cdots + I_n \log_e \frac{1}{D_{1n}}\right)$$
$$+ 2\times10^{-7}\left(I_1 \log_e D + I_2 \log_e D + I_3 \log_e D + \cdots + I_n \log_e D\right)$$
$$= 2\times10^{-7}\left(I_1 \log_e \frac{1}{r_1'} + I_2 \log_e \frac{1}{D_{12}} + I_3 \log_e \frac{1}{D_{13}} + \cdots + I_n \log_e \frac{1}{D_{1n}}\right)$$
$$+ 2\times10^{-7} \log_e D \left(I_1 + I_2 + I_3 + \cdots + I_n\right)$$
$$= 2\times10^{-7}\left(I_1 \log_e \frac{1}{r_1'} + I_2 \log_e \frac{1}{D_{12}} + I_3 \log_e \frac{1}{D_{13}} + \cdots + I_n \log_e \frac{1}{D_{1n}}\right)$$
$$+ 2\times10^{-7} \log_e D\,(0) \qquad \left[\because I_1 + I_2 + I_3 + \cdots + I_n = 0\right]$$

$$\lambda_{1P} = 2\times10^{-7}\left(I_1 \log_e \frac{1}{r_1'} + I_2 \log_e \frac{1}{D_{12}} + I_3 \log_e \frac{1}{D_{13}} + \cdots + I_n \log_e \frac{1}{D_{1n}}\right) \ \frac{\text{wb-turns}}{\text{metre}}$$

Inductance of Composite Conductor Lines :-

→ Composite conductors are conductors composed of two or more strands electrically in parallel. Let strands are identical and share the current equally.

→ Consider a single phase line consisting of two parallel conductors. X and Y. Conductor X consisting of n strands and Conductor Y consisting of m strands as shown in figure



→ Let I = Current flows through conductor X.

−I = Current flows through conductor Y.

→ Assuming uniform current density,

Current carried by each Strand of conductor X =

Current carried by each strand of conductor Y =

Flux linkages of Strand 1 in conductor X is,

$$\lambda_1 = 2\times10^{-7}\left[\frac{I}{n}\log_e\frac{1}{r_1'} + \frac{I}{n}\log_e\frac{1}{D_{12}} + \cdots\cdots + \frac{I}{n}\log_e\frac{1}{D}\right.$$

$$\left. + 2\times10^{-7}\left[\left(-\frac{I}{m}\right)\log_e\frac{1}{D_{11'}} + \left(-\frac{I}{m}\right)\log_e\frac{1}{D_{12'}} + \cdots + \left(-\frac{I}{m}\right)\log_e\right.\right.$$

$$= 2\times10^{-7}\,\frac{I}{n}\left[\log_e\frac{1}{r_1'} + \log_e\frac{1}{D_{12}} + \cdots\cdots + \log_e\frac{1}{D_{1n}}\right]$$

$$- 2\times10^{-7}\,\frac{I}{m}\left[\log_e\frac{1}{D_{11'}} + \log_e\frac{1}{D_{12'}} + \cdots + \log_e\frac{1}{D_{1m'}}\right]$$

$$\lambda_1 = 2\times10^{-7} I \log_e \frac{\sqrt[m]{D_{11'}\, D_{12'}\, D_{13'} \cdots\cdots D_{1m}}}{\sqrt[n]{r_1'\, D_{12}\, D_{13} \cdots\cdots D_{1n}}} \quad \frac{\text{wb-turns}}{\text{metre}}$$

Inductance of strand 1 of conductor 'x' is

$$L_1 = \frac{\text{Flux linkage}}{\text{Current}} = \frac{\lambda_1}{I/n}$$

$$= 2\times10^{-7}\, n \log_e \frac{\sqrt[m]{D_{11'}\, D_{12'}\, D_{13'} \cdots\cdots D_{1m}}}{\sqrt[n]{r_1'\, D_{12}\, D_{13} \cdots\cdots D_{1n}}} \quad \frac{\text{Henrys}}{\text{Metre}}$$

milarly inductance of strand 2 of conductor 'x' is,

$$L_2 = 2\times10^{-7}\, n \log_e \frac{\sqrt[m]{D_{21'}\, D_{22'}\, D_{23'} \cdots\cdots D_{2m}}}{\sqrt[n]{D_{21}\, r_2'\, D_{23} \cdots\cdots D_{2n}}}$$

The average inductance of strands of conductor 'x' is,

$$L_{av} = \frac{L_1 + L_2 + L_3 + \cdots + L_n}{n}$$

But conductor 'x' has 'n' strands which are connected in parallel, so inductance of conductor 'x' is, $\frac{1}{L} = \frac{1}{L_{av}} + \frac{1}{L_{av}} + \frac{1}{L_{av}}$ up to n terms.

$$\Rightarrow \frac{1}{L} = \frac{n}{L_{av}}$$

$$\Rightarrow L = \frac{L_{av}}{n} = \frac{L_1 + L_2 + L_3 + \cdots\cdots + L_n}{n^2}$$

$$\Rightarrow L = 2\times10^{-7}\log_e\left[\frac{\sqrt[mn]{(D_{11'}D_{12'}D_{13'}\cdots D_{1m})(D_{21'}D_{22'}D_{23'}\cdots D_{2m})\cdots\cdots(D_{n1'}D_{n2'}D_{n3'}\cdots}}{\sqrt[n^2]{(r'_1 D_{12}D_{13}\cdots D_{1n})(D_{21}r'_2 D_{23}\cdots D_{2n})\cdots\cdots(D_{n1}D_{n2}D_{n3}\cdots}}\right.$$

Let $r'_1 = D_{11}$, $r'_2 = D_{22}$ and $r'_n = D_{nn}$.

$$\therefore L = 2\times10^{-7}\log_e\left[\frac{\sqrt[mn]{(D_{11'}D_{12'}D_{13'}\cdots D_{1m})(D_{21'}D_{22'}D_{23'}\cdots D_{2m})\cdots\cdots(D_{n1'}D_{n2'}\cdots}}{\sqrt[n^2]{(D_{11}D_{12}D_{13}\cdots D_{1n})(D_{21}D_{22}D_{23}\cdots D_{2n})\cdots\cdots(D_{n1}D_{n2}D_{n3}\cdots}}\right.$$

In the above expression, numerator of Logarithemic function is called GMD (geometric mean distance) and denominator is called GMR (geometric mean readins).

Let GMD represented by D_m and
GMR represented by D_s.

$\therefore$ $L_x = 2\times10^{-7}\log_e\frac{D_M}{D_{sx}}$, for conductor x.

Similarly $L_y = 2\times10^{-7}\log_e\frac{D_M}{D_{sy}}$, for conductor y.

$\therefore$ Loop inductance, $L = L_x + L_y = 2\times10^{-7}\log_e\frac{D_M}{D_{sx}} + 2\times10^{-7}\log_e\frac{D_M}{D_s}$

If two conductors are identical, then $D_{sx} = D_{sy} = D_s$ (say

loop inductance $L = 2\times10^{-7}\log_e\frac{D_M}{D_s} + 2\times10^{-7}\log_e\frac{D_M}{D_s}$

$\Rightarrow \boxed{L = 4\times10^{-7}\log_e\frac{D_M}{D_s}}$ Henrys/metre.

Case 2

Inductance of three phase Lines with equilateral Spacing :-

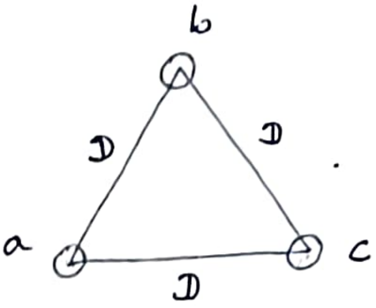


Figure shows a 3-ϕ line with conductors a, b and c spaced at the corners of an equilateral triangle each side 'D'.

Let r = radius of each conductor.

The Currents in the conductors are I_a, I_b and I_c. If we assume balanced 3-ϕ phasor currents then $I_a + I_b + I_c = 0$

Flux linkages of conductor 'a' is,

$$\lambda_a = 2\times10^{-7}\left[I_a \log_e\frac{1}{r'} + I_b \log_e\frac{1}{D} + I_c \log_e\frac{1}{D}\right] \frac{wb-T}{m}$$

$$= 2\times10^{-7}\left[I_a \log_e\frac{1}{r'} + \log_e\frac{1}{D}(I_b + I_c)\right] \frac{wb-T}{m.}$$

$$= 2\times10^{-7}\left[I_a \log_e\frac{1}{r'} + \log_e\frac{1}{D}(-I_a)\right] \quad \left\{\begin{array}{l} \because I_a + I_b + I_c = 0 \\ \Rightarrow I_b + I_c = -I_a \end{array}\right\}$$

$$= 2\times10^{-7} I_a \log_e\frac{D}{r'} \quad \frac{wb\text{-turns}}{metre}$$

Inductance of conductor 'a' is,

$$L_a = \frac{\lambda_a}{I_a} = 2\times10^{-7}\log_e\frac{D}{r'} \quad \frac{Henrys}{metre}$$

= inductance per Phase.

Inductance of three phase lines with Unsymmetrical Spacing :-

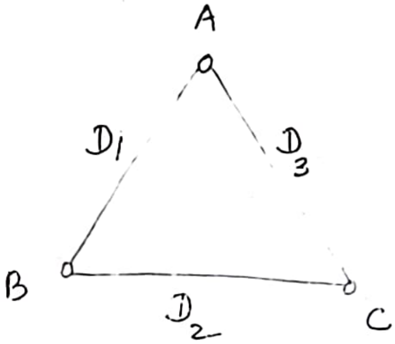
→ When 3-phase line conductors are not equidistant from each other then the conductor spacing is said to be unsymmetrical

Let us consider a 3-phase line with conductors A, B and C, each of radius 'r' metres.

Let D_1, D_2 and D_3 are the distance between the conductors as shown in fig.

I_A, I_B and I_c are the currents flowing through conductors A, B and C respectively.

Assume balanced conditions i.e $I_A + I_B + I_C = 0$



The Line currents are,

$$I_A = I\angle 0 = I \quad (\text{taking } I_A \text{ as a reference phasor})$$

$$I_B = I\angle -120^\circ = I(-0.5 - j0.866)$$

and $$I_C = I\angle -240 = I(-0.5 + j0.866)$$

⟹ The flux linkages of conductor 'A' due to its own current 'I_A' and other conductor currents I_B and

$$\lambda_A = 2\times10^{-7}\left[I_A \log_e \frac{1}{r'} + I_B \log_e \frac{1}{D_1} + I_c \log_e \frac{1}{D_3}\right] \frac{\text{wb-turn}}{\text{metre}}$$

$$= 2\times10^{-7}\left[I \log_e \frac{1}{r'} + I(-0.5 - j0.866)\log_e \frac{1}{D_1} + I(-0.5 + j0.866)\log_e \frac{1}{D_3}\right]$$

$$= 2\times10^{-7}\left[I \log_e \frac{1}{r'} - 0.5 I \log_e \frac{1}{D_1} - 0.5 I \log_e \frac{1}{D_3} + j\, 0.866 I \log_e \frac{1}{D_3} - j0.866 I \log_e \frac{1}{D_1}\right.$$

$$\lambda_A = 2\times10^{-7}\left[I\log_e\frac{1}{r'} - 0.5I\log_e\frac{1}{D_1D_3} + j0.866I\log_e\frac{D_1}{D_3}\right] \frac{\text{wb-turns}}{\text{metre}}$$

$$= 2\times10^{-7}\left[I\log_e\frac{1}{r'} + I\log_e\sqrt{D_1D_3} + j\frac{\sqrt{3}}{2}I\log_e\frac{D_1}{D_3}\right] \frac{\text{wb-turns}}{\text{metre}}$$

$$= 2\times10^{-7}\, I\left[\log_e\frac{1}{r'} + \log_e\sqrt{D_1D_3} + j\sqrt{3}\log_e\sqrt{\frac{D_1}{D_3}}\right] \frac{\text{wb-turns}}{\text{metre}}.$$

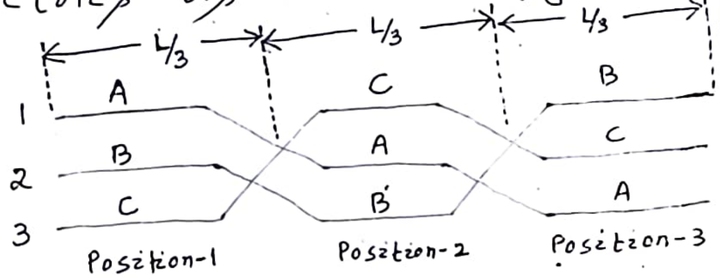
Inductance of conductor 'A' is,

$$L_A = \frac{\lambda_A}{I_A} = \frac{\lambda_A}{I} = 2\times10^{-7}\left[\log_e\frac{1}{r'} + \log_e\sqrt{D_1D_3} + j\sqrt{3}\log_e\sqrt{\frac{D_1}{D_3}}\right] \frac{H}{m}$$

Similarly, $L_B = 2\times10^{-7}\left[\log_e\frac{1}{r'} + \log_e\sqrt{D_1D_2} + j\sqrt{3}\log_e\sqrt{\frac{D_2}{D_1}}\right] \frac{H}{m}$

and $L_C = 2\times10^{-7}\left[\log_e\frac{1}{r'} + \log_e\sqrt{D_2D_3} + j\sqrt{3}\log_e\sqrt{\frac{D_3}{D_2}}\right] \frac{H}{M}$

⟹ From the above it is clear that the inductance of each phase are not same. A different inductance in each phase results in unequal voltage drops in the three phases. Therefore the voltage at the receiving end will not be same for all phases. This unbalancing effect on account of irregular spacing of conductors is avoided by transposition of conductors as shown in figure.



⟹ From figure, the positions of conductors interchanged at regular intervals along the line, so that each conductor occupies the original position of every other conductor over an equal distance. Such an exchange of positions is known as transposition.

→ The effect of transposition is that each cond has the same average inductance.
It is given by,

$$L = \frac{L_A + L_B + L_C}{3}$$

$$= \frac{1}{3}\left[2\times10^{-7}\left(3\log_e\frac{1}{r'} + \log_e\sqrt{D_1D_3} + \log_e\sqrt{D_1D_2} + \log_e\sqrt{D_2D_3} + j\sqrt{3}\log_e\sqrt{\frac{D_1}{D_3}} + j\sqrt{3}\log_e\sqrt{\frac{D_2}{D_1}} + j\sqrt{3}\log_e\sqrt{\frac{D_3}{D_2}}\right)\right]$$

$$= 2\times10^{-7}\left[\log_e\frac{1}{r'} + \frac{1}{3}\left(\log_e D_1D_2D_3\right) + 0\right]$$

$$= 2\times10^{-7}\left[\log_e\frac{1}{r'} + \log_e\sqrt[3]{D_1D_2D_3}\right]$$

$$= 2\times10^{-7}\log_e\frac{\sqrt[3]{D_1D_2D_3}}{r'}\ \frac{\text{Henrys}}{\text{metre}} = 2\times10^{-7}\log_e\frac{D_m}{D_s}\ \frac{\text{Henry}}{\text{met}}$$

= inductance of each conductor, or each Ph

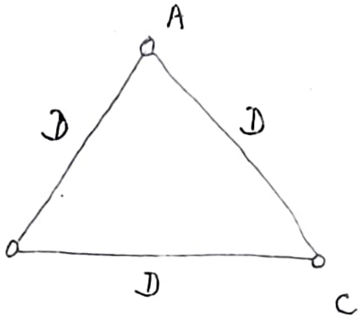
Where $D_m = D_{eq} = \sqrt[3]{D_1D_2D_3}$ and $D_s = r' = GMR$

Case — If the conductors are equispaced
i.e $D_1 = D_2 = D_3 = D$

then inductance of each conductor is,

$$L = 2\times10^{-7}\log_e\frac{\sqrt[3]{DDD}}{r'}$$

$$= 2\times10^{-7}\log_e\frac{D}{r'}$$



Bundled conductors :

A bundle conductor is a conductor made up of two or more sub-conductors and is used as one phase conductor.

Lines of 400KV and higher voltages use bundled conductors.

The sub-conductors of a bundled conductor are separated from each other by a constant distance varying from 0.2 m to 0.6 m depending upon designed voltage and surrounding conditions throughout the length of the line with the help of spacers.

The bundled conductors have filter material or air space inside so that the overall diameter is increased.

The following are the advantages in using bundle conductors:

(i) reduced reactance
(ii) reduced voltage gradient
(iii) reduced corona loss
(iv) reduced radio interference.
(v) reduced surge impedance.

The Procedure for calculating the reactance of bundled conductor is same as for composite conductors.

The basic difference between composite conductor and bundle conductor is that the sub-conductors of a bundle conductor are separated from each other where as the wires of a composite conductor touch each other.

→ Reduced reactance is an important adv of bundling. The reduction of reactance resu from the increased GMR of the bundle.

Let D_s^b = GMR (geometric mean radius) of bundled conductor.

$D_s = r'$ = GMR of individual conductors composing the bundle.

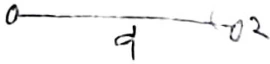
For a two-strand bundle conductor, $D_s^b = \sqrt[4]{(D_s \times d)^2} = \sqrt{}$

For a three-strand bundle conductor, $D_s^b = \sqrt[9]{(D_s \times d \times d)^3} = \sqrt[3]{D_s}$

For a four-strand bunde conductor, $D_s^b = \sqrt[16]{(D_s \times d \times d \times \sqrt{2}d)^4} = 1.$

Note :- We know GMR (i.e D_s) $= \sqrt[n^2]{(D_{11} D_{12} \cdots D_{1n})(D_{21} D_{22} \cdots D_{2n}) \cdots\cdots (D_{n1}}$

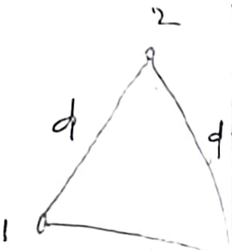
(i) For a two strand bunde $n = 2$



$$\text{so } D_s^b = \sqrt[4]{(D_{11} D_{12})(D_{21} D_{22})} = \sqrt[4]{(r'd)(dr')}$$

$$= \sqrt[4]{(r'd)^2} = \sqrt[4]{(D_s d)^2} = \sqrt{D_s \times d}$$

(ii) For a three-strand bunde $n = 3$.



$$\text{so } D_s^b = \sqrt[9]{(D_{11} D_{12} D_{13})(D_{21} D_{22} D_{23})(D_{31} D_{32} D_{33})}$$

$$= \sqrt[9]{(r'dd)(dr'd)(ddr')}$$

$$= \sqrt[9]{(r'dd)^3} = \sqrt[9]{(D_s dd)^3} = \sqrt[3]{D_s d^2}$$

(iii) For a four-strand bunde conductor $n = 4$

$$\text{so } D_s^b = \sqrt[16]{(D_{11} D_{12} D_{13} D_{14})(D_{21} D_{22} D_{23} D_{24})(D_{31} D_{32} D_{33} D_{34})(D_{41} D_{42} D_{43} D_{44})}$$

$$= \sqrt[16]{(r'd\ \sqrt{2}d\ d)(dr'd\sqrt{2}d)(\sqrt{2}d\, d\, r'd)(d\sqrt{2}d\, d\, r')}$$

$$= \sqrt[16]{(r'dd\sqrt{2}d)^4} = \sqrt[16]{(D_s dd\sqrt{2}d)^4}$$

$$= 1.09 \sqrt[4]{D_s d^3} \quad]$$

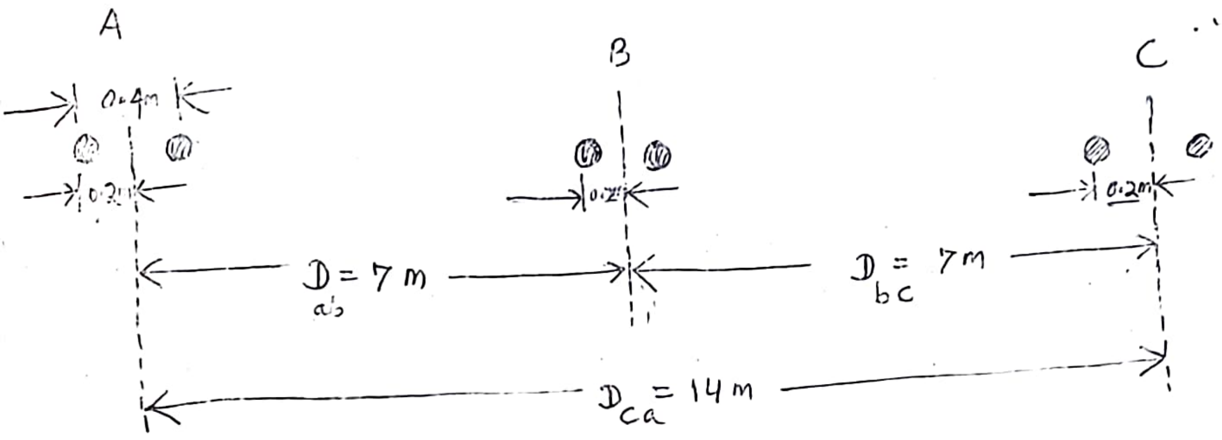
> The inductance of the bundled conductor is $L = 2\times10^{-7} \log_e \frac{D_m}{D_s^b}$ $\frac{H}{m}$

where $D_M = D_{eq}$ = GMD of a bundled conductor

GMD can be determined by taking the root of the product of distances from centre of one bundle to the centres of other bundles.

~~roblem~~ :- Find the inductive reactance of a 3-phase bundled conductor line with 2 conductors per phase with spacing of 40 cm. phase to phase separation is 7 m in horizontal configuration. All conductors are ACSR with dia of 3.5 cm.

lution :-



= radius of each subconductr $= \frac{3.5}{2} = 1.75$ cm $= 1.75\times10^{-2}$ m.

Spacing between subconductors of one phase $d = 0.4$ m

GMR of individual conductors composing the bundle is,

$D_s = r' = 1.363\times10^{-2}$ m. ~~$= 0.7788 \times 1.75 \times 10^{-2}$~~

GMR of bundled conductor $= D_s^b = \sqrt{D_s \times d} = \sqrt{1.363\times10^{-2}\times0.4}$

$= 0.0738$ m

→ $D_{eq} = D_M$ = GMD of bundled conductore

$= \sqrt[3]{D_{ab} D_{bc} D_{ca}} = \sqrt[3]{7 \times 7 \times 14} = 8.819\ M$

∴ Inductance of the bundled conductore,

$L = 0.2 \times 10^{-7} \log_e \frac{D_M}{D_s^b}$ H/m

$= 2 \times 10^{-7} \log_e \frac{8.819}{0.0738}$ H/M $= 9.566 \times 10^{-7}$ H/m.

∴ Inductive reactance, $X_L = 2\pi f L = 2\pi \times 50 \times 9.566 \times 10^{-7}$

$= 3.0039 \times 10^{-4}$ ohm.

Inductance of a 3-phase double ckt line with symmetrical spacing :—

→ Let us considere a 3-ϕ double ckt connected in parallel conductors A, B, C forming one ckt and conductors A′ B′ C′ forming othere ckt as shown in figure.

→ Flux linkages of phase A conductors,

$\lambda_A = 2 \times 10^{-7} \left[I_A \left(\log_e \frac{1}{r'} + \log_e \frac{1}{2d} \right) + I_B \left(\log_e \frac{1}{d} + \log_e \frac{1}{\sqrt{3} d} \right) + I_C \left(\log_e \frac{1}{\sqrt{3} d} \right.\right.$

$= 2 \times 10^{-7} \left[I_A \log_e \frac{1}{2dr'} + I_B \log_e \frac{1}{\sqrt{3} d^2} + I_C \log_e \frac{1}{\sqrt{3} d^2} \right]$

$= 2 \times 10^{-7} \left[I_A \log_e \frac{1}{2dr'} + \log_e \frac{1}{\sqrt{3} d^2} (I_B + I_C) \right]$

$= 2 \times 10^{-7} \left[I_A \log_e \frac{1}{2dr'} - I_A \log_e \frac{1}{\sqrt{3} d^2} \right]$ $\left\{ \because I_A + I_B + I_C = 0 \Rightarrow I_B + I_C = -I_A \right\}$

$= 2 \times 10^{-7} I_A \log_e \frac{\sqrt{3}\, d}{2 r'}$ wb-turns/meter.

∴ Inductance of conductor A is,

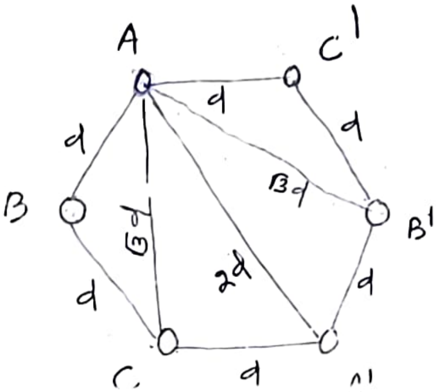
$$L_A = \frac{\lambda_A}{I_A} = 2\times10^{-7} \log_e \frac{\sqrt{3}\,d}{2r'} \quad \frac{\text{Henrys}}{\text{metre}}$$

But A and A' are conductors connected in parallel forming one phase.

∴ Inductance of each phase

$$= \frac{1}{2} L_A$$

$$= 10^{-7} \log_e \frac{\sqrt{3}\,d}{2r'} \ H/m.$$

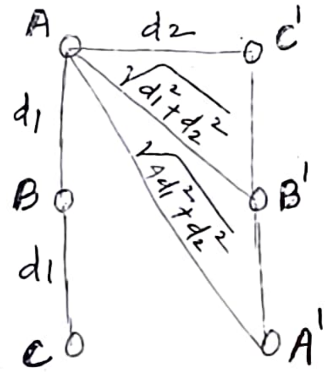


= [illegible] = $\sqrt{3}$ × ([illegible])

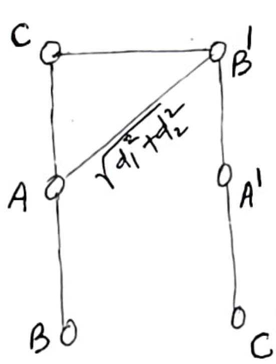
Inductance of a 3-phase double circuit with unsymmetrical spacing (Transposed Lines) :-

→ Let us consider a 3-φ double ckt connected in parallel conductors A, B, C forming one circuit and conductors A', B', C' forming the other.

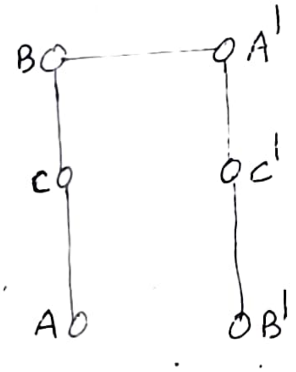
→ Figure shows the conductors unsymmetrically spaced & transposed.



Position-1



Position-2



Position-3

→ Flux linkages with conductor 'A' in Position-1 is

$$\lambda_{A1} = 2\times10^{-7}\left[I_A\left(\log_e\frac{1}{r'}+\log_e\frac{1}{\sqrt{4d_1^2+d_2^2}}\right)+I_B\left(\log_e\frac{1}{d_1}+\log_e\frac{1}{\sqrt{d_1}}\right.\right.$$
$$\left.+I_C\left(\log_e\frac{1}{2d_1}+\log_e\frac{1}{d_2}\right)\right]$$

Flux linkages with conductor 'A' in Position-2 is,

$$\lambda_{A2} = 2\times10^{-7}\left[I_A\left(\log_e\frac{1}{r'}+\log_e\frac{1}{d_2}\right)+I_B\left(\log_e\frac{1}{d_1}+\log_e\frac{1}{\sqrt{d_1^2+d_2^2}}\right)\right.$$
$$\left.+I_C\left(\log_e\frac{1}{d_1}+\log_e\frac{1}{\sqrt{d_1^2+d_2^2}}\right)\right]$$

Flux linkages with conductor 'A' in Position-3 is,

$$\lambda_{A3} = 2\times10^{-7}\left[I_A\left(\log_e\frac{1}{r'}+\log_e\frac{1}{\sqrt{4d_1^2+d_2^2}}\right)+I_B\left(\log_e\frac{1}{2d_1}+\log_e\frac{1}{d_2}\right)\right.$$
$$+I_C\left(\log_e\frac{1}{d_1}+\log_e\frac{1}{\sqrt{d_1^2+d_2^2}}\right)$$

∴ Average Flux Linkages with conductor 'A' is

$$\lambda_A = \frac{\lambda_{A1}+\lambda_{A2}+\lambda_{A3}}{3} = \frac{2\times10^{-7}}{3}\,I_A\log_e\frac{2d_1^3\cdot d_2\cdot(d_1^2+}{(r')^3 d_2(4d_1^2+}$$

$$\Rightarrow \lambda_A = 2\times10^{-7}\,I_A\log_e\frac{\left[2d_1^3\cdot d_2\cdot(d_1^2+d_2^2)\right]^{\frac{1}{3}}}{\left[(r')^3 d_2(4d_1^2+d_2^2)\right]^{\frac{1}{3}}}$$

$$\Rightarrow \lambda_A = 2\times10^{-7}\,I_A\log_e\frac{2^{\frac{1}{3}}\cdot d_1\cdot(d_1^2+d_2^2)^{\frac{1}{3}}}{r'\cdot(4d_1^2+d_2^2)^{1/3}}$$ wb-turns/mete.

Inductance of conductor A is,

$$L_A = \frac{\tau_A}{I_A} = 2\times10^{-7}\log_e \frac{2^{1/3} d_1 (d_1^2+d_2^2)^{\frac{1}{3}}}{r'(4d_1^2+d_2^2)^{1/3}} \; H/M.$$

ince conductors are electrically in Parallel, inductance of each phase is,

$$L = \frac{1}{2}L_A = \frac{1}{2}\left[2\times10^{-7}\log_e \frac{2^{\frac{1}{3}} d_1 (d_1^2+d_2^2)^{\frac{1}{3}}}{r'(4d_1^2+d_2^2)^{\frac{1}{3}}}\right]$$

$$\Rightarrow L = 2\times10^{-7}\log_e \left[\frac{2^{\frac{1}{3}}\cdot d_1 (d_1^2+d_2^2)^{\frac{1}{3}}}{r'(4d_1^2+d_2^2)^{\frac{1}{3}}}\right]^{\frac{1}{2}}$$

$$\Rightarrow L = 2\times10^{-7}\log_e 2^{\frac{1}{6}}\cdot\left(\frac{d_1}{r'}\right)^{\frac{1}{2}}\left(\frac{d_1^2+d_2^2}{4d_1^2+d_2^2}\right)^{\frac{1}{6}} \quad \frac{H}{M}$$

ase :- If d_2 is very large than d_1 then

$$\frac{d_1^2+d_2^2}{4d_1^2+d_2^2} \longrightarrow 1.$$

$$\text{So, } L = 2\times10^{-7}\log_e 2^{\frac{1}{6}}\left(\frac{d_1}{r'}\right)^{\frac{1}{2}} \quad \frac{H}{M}$$

Skin Effect :-

→ When d.c flows in the [illegible] conductor, the current is uniformly distributed across the cross-section of the conductor. whereas When ac flows in the conductor, the current does distribute uniformly, it concentrate near the surface of the conductor.

→ The tendency of alternating current to concentrate near the surface of a conductor is known as skin effect.

→ Due to skin effect, the effective area of cross-Se of the conductor through which a.c flows in reduced. When area decreases, then resistance of the conductor increases.

So ac resistance is greater than dc resistance.

→ The cause of skin effect can be explained by the following example.

Let us consider a solid conductor consisting of large number of strands. Each strand carrying a small part of the alternating current. The inductance of each strand will vary according to the position. The strands near the centre are surrounded by a large magnetic flux, so they hav large inductance (as $L = \phi/I$). This large inductan produces high reactance. The high reactance of inner strands causes the alternating current to flow near the surface of conductor. This crowding of current near the conductor surface is the skin effect.

C.L. WADHWA

(33)

Problem ① Determine the self GMD of the following types of conductors in terms of radius 'r' of an individual strand.

Ans: Self GMD = D_s^b

$$D_s^b = \sqrt[16]{(r' D_{12} D_{13} D_{14})(D_{21}\, r' D_{23} D_{24})(D_{31} D_{32}\, r' D_{34})(D_{41} D_{42} D_{43} r')}$$

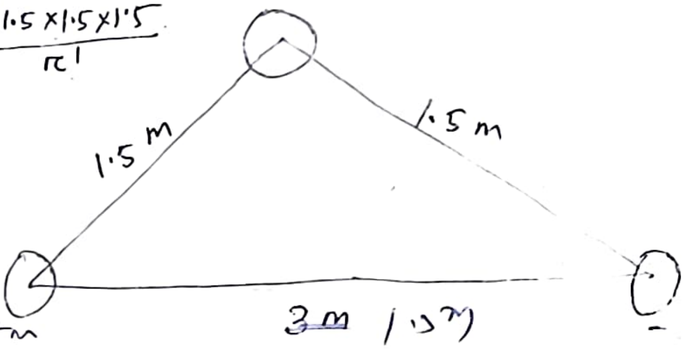
$$= \sqrt[16]{(r')^4 (2r)(2\sqrt{2}r)(2r)(2r)(2r)(2\sqrt{2}r)(2\sqrt{2}r)(2r)(2r)(2r)(2\sqrt{2}r)(2r)}$$

$$= \sqrt[16]{(0.7788r)^4 (2r)^8 (2\sqrt{2}r)^4}$$

$$= 1.7228\, r.$$

Problem-2 A single ckt 3-ϕ line operated at 50 Hz is arranged as follows. The conductor diameter is 0.6 cm. Determine the inductance and inductive reactance per Km.

Ans:- Inductance, $L = 2\times10^{-7} \log_e \frac{\sqrt[3]{1.5\times1.5\times1.5}}{r'}$



$\Rightarrow L = 2\times10^{-7} \log_e \frac{1.5}{r(0.7788)}$ $r e^{\frac{1}{4}}$ $\frac{1}{0.7788}$

$\Rightarrow L = 1.292 \frac{mH}{Km}$

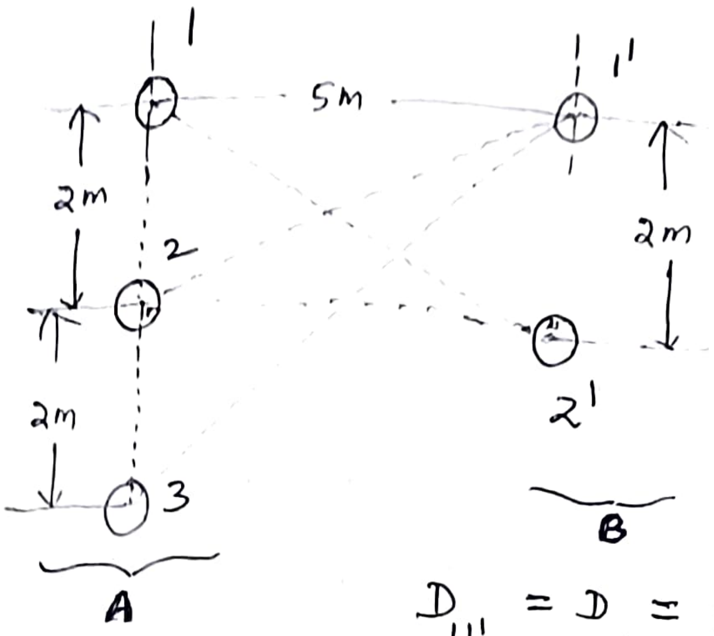
$r = 0.3 cm = 0.3\times10^{-2} m$

∴ inductive reactance $X_L = 2\pi f L = 2\pi\times50\times1.292\times10^{-3}$

$= 0.4 \frac{\Omega}{Km}.$

Problem-3 Determine the inductance of a 1-Phase transmission line having the following arrangement of conductors. One circuit consists of three wires of 2mm diameter each and the other circuit two wires of 4mm dia each.

Ans:-



For line A:-

$D_{11} = D_{22} = D_{33} = r' = 0.778r$
$= 0.7788 \times 10^{-3}$ m

$D_{12} = D_{21} = D_{23} = D_{32} = 2$

$D_{13} = D_{31} = 4$

$D_{11'} = D_{22'} = 5$ m

$D_{21'} = D_{12'} = D_{32'} = \sqrt{5^2+2^2} = 5.3$

$D_{31'} = \sqrt{5^2+4^2} = 6.403$ m

Inductance of line A $= L_A = 2\times10^{-7} \log_e \dfrac{\sqrt[mn]{(D_{11'}\,D_{12'})(D_{21'}\,D_{22'})(D_{31'}\,D_{3}}}{\sqrt[n^2]{(D_{11}\,D_{12}\,D_{13})(D_{21}\,D_{22}\,D_{23})(D_{3}}}$

Here m = 2 and n = 3.

$\therefore L_A = 2\times10^{-7} \log_e \dfrac{\sqrt[6]{5\times5.385\times5.385\times5\times6.403\times5.385}}{\sqrt[9]{(0.7788\times10^{-3})^3 \times 2\times4\times2\times2\times4\times2}}$

$= 6.91\times10^{-7} \dfrac{H}{m} = 0.691 \dfrac{mH}{Km}$.

For line B:- $D_{1'1} = 5$ m $= D_{2'2}$ $D_{1'3} = 6.403$ m

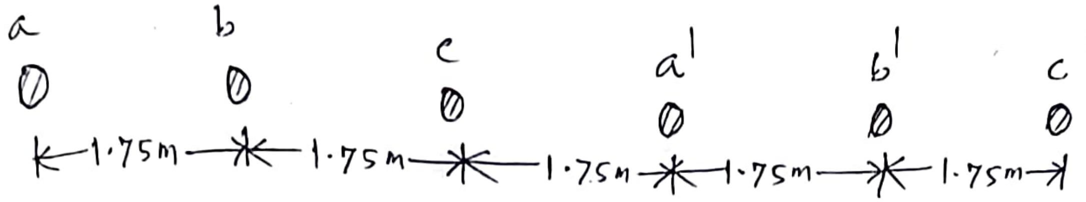
$D_{2'3} = D_{1'2} = D_{2'1} = 5.385$ m

Here m = 3 and n = 2

Inductance of line B $= L_B = 2\times10^{-7} \log_e \dfrac{\sqrt[6]{(D_{1'1}\,D_{1'2}\,D_{1'3})(D_{2'1}\,D_{2'2}\,D_{2'3})}}{\sqrt[4]{(D_{1'1'}\,D_{1'2'})(D_{2'1'}\,D_{2'2'})}} = 0.91$

Total inductance $= L_A + L_B = 0.691 + 0.914 = 1.605 \dfrac{mH}{Km}$

Problem-4: Determine the inductance per km/phase of a double circuit 3-ϕ line. The radius of each conductor is 15 mm.



$ab = bc = d_1 = 1.75$ m.

and $ac' = d_2 = 8.75$ m.

$r' = 0.7788r = 0.7788 \times 15 \times 10^{-3} = 0.01168$ m

∴ Inductance of each phase

$$L = 2\times10^{-7} \log_e (2)^{\frac{1}{6}} \left(\frac{d_1}{r'}\right)^{\frac{1}{2}} \left(\frac{d_1^2+d_2^2}{4d_1^2+d_2^2}\right)^{\frac{1}{6}} \frac{H}{m}$$

$$= 2\times10^{-7} \log_e 2^{\frac{1}{6}} \cdot \left(\frac{1.75}{0.01168}\right)^{\frac{1}{2}} \left(\frac{1.75^2+8.75^2}{4\times1.75^2+8.75^2}\right)$$

$$= 0.52 \frac{mH}{km.}$$

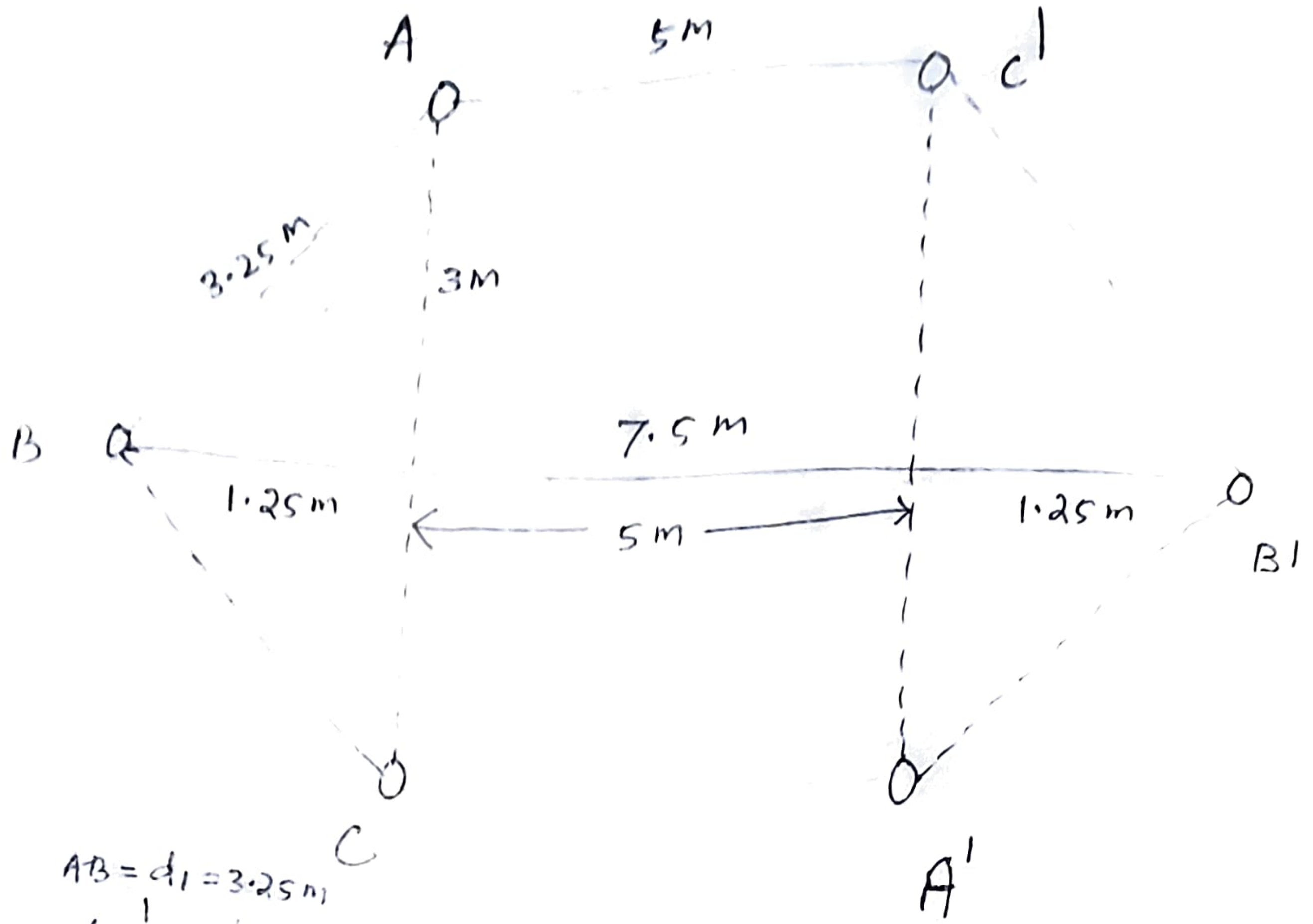
Problem-5 Determine the inductance per km of a double circuit 3-ϕ line as shown in fig. The transmission line is transposed within each circuit and each circuit remains on its own side. The diameter of each conductor is 15 mm.

Ans:- radius of each conductor, $r = \frac{15}{2} = 7.5$ mm $= 7.5\times10^{-3}$ m

∴ $r' = 0.7788r$

$= 0.7788(7.5\times10^{-3})$

$= 5.841\times10^{-3}$ m.



$AB = d_1 = 3.25 m$

$AC' = d_2 = 5 m$

Inductance Per phase $L = 2 \times 10^{-7} \log_e (2)^{\frac{1}{6}} \left(\frac{d_1}{r'}\right)^{\frac{1}{2}} \left(\frac{d_1^2 + d_2^2}{4d_1^2 + d_2^2}\right)^{\frac{1}{6}}$

$= 0.63392 \frac{mH}{Km}$.

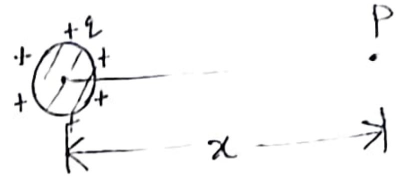
Electric Field of a Long straight conductor :

→ Electric field is a space surrounding a charge within which there is a force acting on the test charge.

Let us consider a long straight cylindrical conductor of radius 'r' and having a charge 'q' coulombs per metre of its length.

It has an electric field.

'p' is any point in the field at distance x from centre of conductor at which electric intensity is to be calculated.



→ Imagine a cylindrical surface of radius 'x' and length one metre.

$\therefore$ Area $= (2\pi x)(1) = 2\pi x \ m^2$

$\boxed{Area = 2\pi r l}$

electric flux density is $B = D = \frac{Flux}{Area} = \frac{Charge}{Area}$

According to Gauss law, charge in a closed surface equals to the flux emerging from the surface

$$\Rightarrow D = \frac{q}{2\pi x} \ C/m^2$$

→ Electric intensity, $E = \frac{D}{\epsilon} = \frac{D}{\epsilon_0 \epsilon_r} = \frac{D}{\epsilon_0}$ $\left[\because \epsilon_r = 1, \text{ for air}\right.$

$$\Rightarrow E = \frac{q}{2\pi\epsilon_0 x} \ V/m$$

This is the expression for electric intensity at a distance 'x' metre from the centre of the conductor.

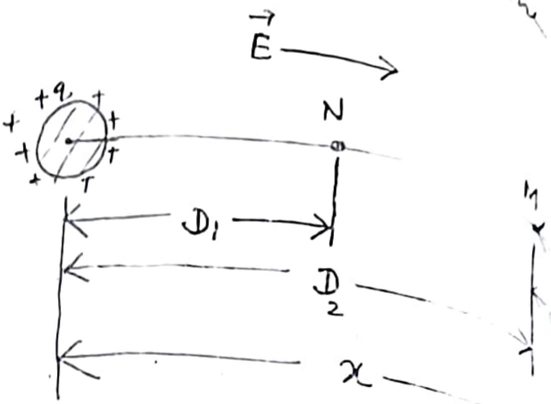
The Potential difference between two Points due to a charge :

→ Let us consider a long straight cylindrical Condu of radius 'r' and having a charge q Cou per metre of its length.

→ The electric intensity at a distance 'x' from the centre of the conductor is,

$$E = \frac{q}{2\pi \epsilon_0 x} \; V/m$$

→ The Potential at any Point in the electric field is defined as the amount of work done Per unit positive charge when taken from infinity to that point.



The Potential difference between two Points distances D_1 and D_2 is defined as the amount work done in moving a unit Positive cha from D_2 to D_1 as shown in fig.

→ Let a test charge 'q_0' is placed at Point 'P'. Force acting on test charge is, $F = Eq_0$
The direction of force is along the direction of electric intensity.
If test charge moves through a small distance against the direction of force then small work is be done. It is given by,

$$dW = \vec{F}\cdot\vec{dx} = (F)(dx)\cos 180 = -F\,dx = -Eq_0\,dx$$

$$\Rightarrow dW = -\frac{qq_0}{2\pi\epsilon_0 x}\,dx$$

$$\Rightarrow \frac{dW}{q_0} = \frac{-q}{2\pi\epsilon_0 x}\cdot dx$$

$$\Rightarrow dV = \frac{-q}{2\pi\epsilon_0 x}\cdot dx$$

$$V = \int \frac{q}{2\pi\epsilon_0 x}\,dx \qquad = \frac{q}{2\pi\epsilon_0}\int_{D_2}^{D_1}\frac{dx}{x}$$

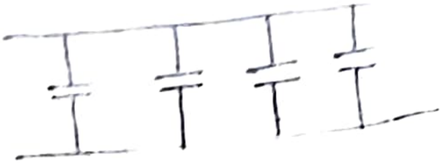
$$V = \frac{q}{2\pi\epsilon_0}\int_{D_1}^{D_2}\frac{dx}{x} \qquad \frac{q}{2\pi\epsilon_0}\left[\log_e x\right]_{D_1}^{D_2}$$

$$\boxed{V = \frac{q}{2\pi\epsilon_0}\log_e\frac{D_2}{D_1}}$$

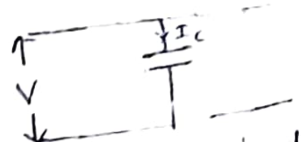
apacitance of a transmission line :

Any two conductors separated by a dielectric constitute a capacitor. An overhead line, two conductors form two plates of a capacitor and air medium b/n them behaves as dielectric

The capacitance of overhead line is uniformly distributed over the total length of the line and may be regarded as a series of capacitors connected between the conductors as shown in figure.



When an alternating p.d is applied across a transmission line, it draws a leading current, even when supplying no load. This leading current is called charging current and is in quadrature with the applied voltage.



Let V = Voltage applied.
C = Capacitance betn the lines.
f = frequency of supply.
I_c = charging current

$$\therefore \quad I_c = V/X_c = V\Big/\frac{1}{2\pi f C} = 2\pi f C V$$

$$V - \int_{D_2}^{D_1} \frac{q}{2\pi\epsilon_0 x} \cdot dx = -\frac{q}{2\pi\epsilon_0} \int_{D_2}^{D_1} \frac{dx}{x}$$

$$V = \frac{q}{2\pi\epsilon_0} \int_{D_1}^{D_2} \frac{dx}{x} = \frac{q}{2\pi\epsilon_0} \left[\log_e x \right]_{D_1}^{D_2}$$

$$\boxed{V = \frac{q}{2\pi\epsilon_0} \log_e \frac{D_2}{D_1}}$$

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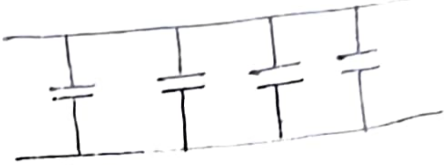

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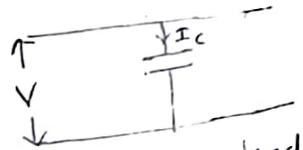


Let V = Voltage applied.

C = Capacitance betⁿ the lines.

f = frequency of supply.

I_c = charging current

$$\therefore \quad I_c = V/X_c = V/\frac{1}{2\pi f C} = 2\pi f C V$$

→ If the capacitance of the line is high then it draws more charging current which compensates the lagging component of load current. As a resul the resultant current flowing in the line reduces. This reduction of resultant current produces reduction in line losses. So transmission efficiency increases. Also reduction of resultant current produces reducti in voltage drop, which improve voltage regulation.

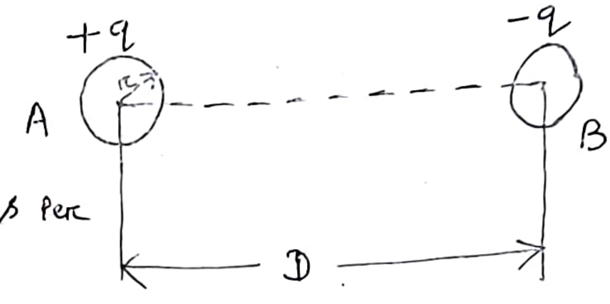
High capacitance of transmission line improves the power factor.

Capacitance of a two-wire line :-

→ Let us consider a single phase overhead line consisting of two parallel conductors A and B separated by 'D' metres apart in air as shown in figure.

→ Let r = radius of each conductor.

+q and -q are two charges per length of two conductors A and B respectively.



The P.d between conductor 'A' and neutral infinite Plan is

$$V_A = \int_r^{\infty} \frac{q}{2\pi\epsilon_0 x} \cdot dx + \int_D^{\infty} \frac{-q}{2\pi\epsilon_0 x} \cdot dx$$

$$= \frac{q}{2\pi\epsilon_0} \log_e\left(\frac{\infty}{r}\right) - \frac{q}{2\pi\epsilon_0} \log_e\left(\frac{\infty}{D}\right)$$

$$= \frac{q}{2\pi\epsilon_0} \log_e \frac{D}{r}$$

larly P.d between conductor 'B' and neutral infinite ine is,

$$V_B = \int_{r}^{\infty} -q/_{2\pi\epsilon_0 x} \cdot dx + \int_{D}^{\infty} q/_{2\pi\epsilon_0 x} \cdot dx$$

$$= -\frac{q}{2\pi\epsilon_0}\log_e\left(\frac{\infty}{r}\right) + \frac{q}{2\pi\epsilon_0}\log_e\left(\frac{\infty}{D}\right)$$

$$= \frac{q}{2\pi\epsilon_0}\log_e\frac{r}{D}$$

$$= -\frac{q}{2\pi\epsilon_0}\log_e\frac{D}{r}.$$

Potential difference between conductors A and B is

$$V_{AB} = V_A - V_B = \frac{q}{2\pi\epsilon_0}\log_e\frac{D}{r} - \left(-\frac{q}{2\pi\epsilon_0}\log_e\frac{D}{r}\right)$$

$$\Rightarrow V_{AB} = \frac{2q}{2\pi\epsilon_0}\log_e\frac{D}{r} = \frac{q}{\pi\epsilon_0}\log_e\frac{D}{r}$$

∴ Capacitance, $C_{AB} = \frac{q}{V_{AB}} = \frac{q}{\frac{q}{\pi\epsilon_0}\log_e\frac{D}{r}}$

$$\Rightarrow \boxed{C_{AB} = \frac{\pi\epsilon_0}{\log_e\frac{D}{r}}} \text{ Farad/m.}$$

$\epsilon_0 = 8.85 \times 10^{-12}$

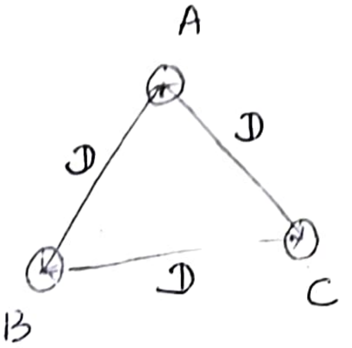
$\mu_0 = 4\pi \times 10^{-7}$

Capacitance of a 3-Phase line with equilateral s

→ Let us consider a 3-phase line with conduc A, B and C spaced at the equilateral triang each side D as shown in figure.

→ r = radius of each conductor.

q_A, q_B and q_C are three charges per metre length of three conductors A, B and C respectively.



→ P.d between conductor 'A' and infinite neutral plan is

$$V_A = \int_r^{\infty} \frac{q_A}{2\pi\epsilon_0 x}\,dx + \int_D^{\infty} \frac{q_B}{2\pi\epsilon_0 x}\,dx + \int_D^{\infty} \frac{q_C}{2\pi\epsilon_0 x}\,dx$$

$$= \frac{q_A}{2\pi\epsilon_0}\log_e\left(\frac{\infty}{r}\right) + \frac{q_B}{2\pi\epsilon_0}\log_e\left(\frac{\infty}{D}\right) + \frac{q_C}{2\pi\epsilon_0}\log_e\left(\frac{\infty}{D}\right)$$

$$= \frac{q_A}{2\pi\epsilon_0}\log_e\left(\frac{\infty}{r}\right) + \frac{1}{2\pi\epsilon_0}\log_e\left(\frac{\infty}{D}\right)\left[q_B + q_C\right]$$

$$= \frac{q_A}{2\pi\epsilon_0}\log_e\left(\frac{\infty}{r}\right) + \frac{1}{2\pi\epsilon_0}\log_e\left(\frac{\infty}{D}\right)\left[-q_A\right]$$

For balance supply $q_A + q_B + q_C = 0$ $\Rightarrow q_B + q_C = -q_A$

$$= \frac{q_A}{2\pi\epsilon_0}\log_e\left(\frac{\infty}{r}\right) - \frac{q_A}{2\pi\epsilon_0}\log_e\left(\frac{\infty}{D}\right)$$

$$= \frac{q_A}{2\pi\epsilon_0}\log_e\frac{D}{r}$$

∴ Capacitance of conductor 'A' with respect to neutral is

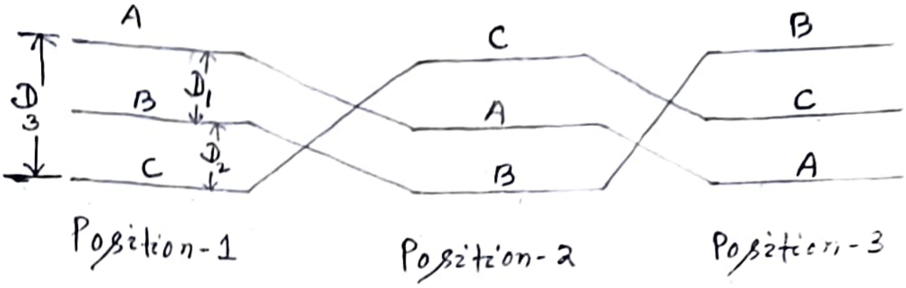
$$C_A = \frac{q_A}{V_A} = \frac{q_A}{\frac{q_A}{2\pi\epsilon_0}\log_e\frac{D}{r}} = \frac{2\pi\epsilon_0}{\log_e\frac{D}{r}} \text{ F/m.}$$

The expressions C_B and C_C are same as C_A.

Capacitance of a 3-phase line with unsymmetrical spacing :- (Transposed)

→ Let us consider a 3-phase transposed line having unsymmetrical spacing.

→ Assume balanced conditions i.e $q_A + q_B + q_C = 0$



→ Considering all the three sections of the transposed line for phase A,

Potential of conductor 'A' in Position-1 (with respect to neutral infinite plane) is

$$V_1 = \int_{r}^{\infty} \frac{q_A}{2\pi\epsilon_0 x} \cdot dx + \int_{D_1}^{\infty} \frac{q_B}{2\pi\epsilon_0 x} \cdot dx + \int_{D_3}^{\infty} \frac{q_C}{2\pi\epsilon_0 x} \cdot dx$$

$$= \frac{1}{2\pi\epsilon_0}\left[q_A \log_e \infty - q_A \log_e r + q_B \log_e \infty - q_B \log_e D_1 + q_C \log_e \infty - q_C \log_e D_3\right]$$

$$= \frac{1}{2\pi\epsilon_0}\left[q_A \log_e \frac{1}{r} + q_B \log_e \frac{1}{D_1} + q_C \log_e \frac{1}{D_3} + (q_A + q_B + q_C)\log_e \infty\right]$$

$$= \frac{1}{2\pi\epsilon_0}\left[q_A \log_e \frac{1}{r} + q_B \log_e \frac{1}{D_1} + q_C \log_e \frac{1}{D_3}\right]$$

Potential of conductor 'A' in Position-2 (w.r.t neutral infinite plane) is

$$V_2 = \int_{r}^{\infty} \frac{q_A}{2\pi\epsilon_0 x} \cdot dx + \int_{D_2}^{\infty} \frac{q_B}{2\pi\epsilon_0 x} \cdot dx + \int_{D_1}^{\infty} \frac{q_C}{2\pi\epsilon_0 x} dx.$$

$$= \frac{1}{2\pi\epsilon_0}\left[q_A \log_e \frac{1}{r} + q_B \log_e \frac{1}{D_2} + q_C \log_e \frac{1}{D_1}\right]$$

Potential of conductor 'A' in Position-3 (w.r.t neutral infinite Plane)

$$V_3 = \int_{r}^{\infty} \frac{q_A}{2\pi\epsilon_0 x} \cdot dx + \int_{D_3}^{\infty} \frac{q_B}{2\pi\epsilon_0 x} \cdot dx + \int_{D_2}^{\infty} \frac{q_c}{2\pi\epsilon_0 x} \cdot dx$$

$$= \frac{1}{2\pi\epsilon_0} \left[q_A \log_e \frac{1}{r} + q_B \log_e \frac{1}{D_3} + q_c \log_e \frac{1}{D_2} \right]$$

∴ Average value of voltage of conductor 'A'

$$V_A = \frac{V_1 + V_2 + V_3}{3} = \frac{1}{6\pi\epsilon_0} \left[q_A \log_e \frac{1}{r^3} + q_B \log_e \frac{1}{D_1 D_2 D_3} \right.$$

$$q_c \log_e \frac{1}{D_1 D_2 D}$$

$$= \frac{1}{6\pi\epsilon_0} \left[q_A \log_e \frac{1}{r^3} + (q_B + q_c) \log_e \frac{1}{D_1 D_2 D_3} \right]$$

$$= \frac{1}{6\pi\epsilon_0} \left[q_A \log_e \frac{1}{r^3} - q_A \log_e \frac{1}{D_1 D_2 D_3} \right]$$

$$= \frac{1}{2\pi\epsilon_0} \times \frac{1}{3} \left[q_A \log_e \frac{1}{r^3} - q_A \log_e \frac{1}{D_1 D_2 D_3} \right]$$

$$= \frac{1}{2\pi\epsilon_0} \left[q_A \log_e \left(\frac{1}{r^3}\right)^{\frac{1}{3}} - q_A \log_e \left(\frac{1}{D_1 D_2 D_3}\right)^{\frac{1}{3}} \right]$$

$$V_A = \frac{1}{2\pi\epsilon_0} \cdot q_A \cdot \log_e \frac{(D_1 D_2 D_3)^{\frac{1}{3}}}{r} \text{ volts.}$$

∴ Capacitance of conductor 'A' $= C_A = \frac{q_A}{V_A} = \frac{2\pi\epsilon_0}{\log_e \frac{(D_1 D_2 D}{r}}$

Similarly C_B and C_c are obtained which is same as C_A. $\frac{\text{Far}}{\text{me}}$

Effect of earth on the Capacitance of 1-ϕ overhead

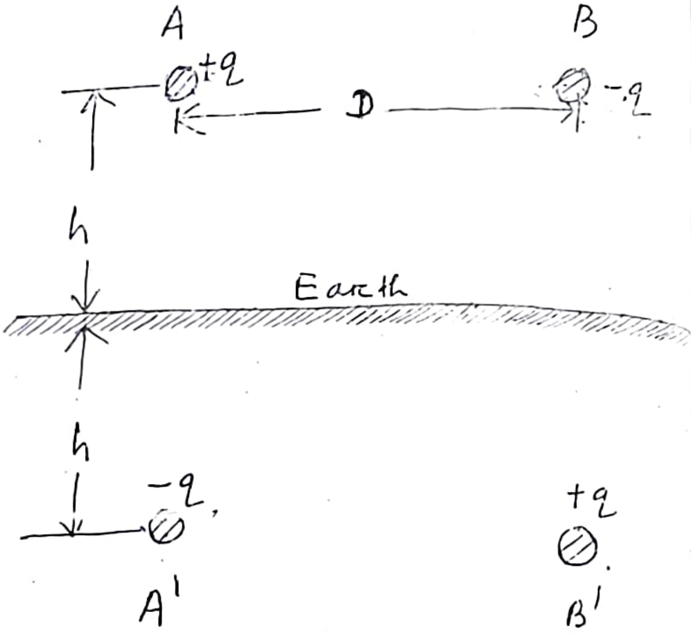
→ The Presence of earth alters the electric field of line and so affects its capacitance.

→ The electric flux lines and equipotential lines are orthogonal to each other.

→ The earth is considered to be conducting and an equipotential plane of infinite extent. Therefore the electric flux lines are forced to cut the earth surface orthogonally.

→ Consider a 1-ϕ overhead line having two conductors A and B placed at a height 'h' from earth surface. +q coulombs per metre length be the charge of conductor and -q coulomb's per metre length be the charge of conductor

The earth is considered to be zero potential. It is possible only if there is an image conductor having a charge -q coulombs per metre at a depth below the earth and a conductor B' having a charge +q coulombs per metre at a depth h below the earth as shown in figure.



Potential of conductor 'A' is,

$$= \int_{r}^{\infty} \frac{q}{2\pi\epsilon_0 x} \cdot dx + \int_{D}^{\infty} \frac{-q}{2\pi\epsilon_0 x} \cdot dx$$

$$+ \int_{2h}^{\infty} \frac{-q}{2\pi\epsilon_0 x} \cdot dx + \int_{\sqrt{4h^2+D^2}}^{\infty} \frac{q}{2\pi\epsilon_0 x} \cdot dx$$

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$$V_A = \frac{q}{2\pi\epsilon_0}\left[\log_e \frac{1}{r} + \log_e D + \log_e 2h - \log_e \sqrt{4h^2+D^2}\right]$$

$$= \frac{q}{2\pi\epsilon_0} \log_e \frac{2hD}{r\sqrt{4h^2+D^2}}$$

Potential of conductor 'B' is,

$$V_B = \int_r^{\infty} \frac{-q}{2\pi\epsilon_0 x}\cdot dx + \int_D^{\infty} \frac{q}{2\pi\epsilon_0 x}\cdot dx + \int_{2h}^{\infty} \frac{q}{2\pi\epsilon_0 x}\cdot dx + \int_{\sqrt{4h^2+d^2}}^{\infty} \frac{-q}{2\pi\epsilon_0 x}\cdot dx$$

$$= -\frac{q}{2\pi\epsilon_0} \log_e \frac{2hD}{r\sqrt{4h^2+D^2}}$$

∴ P.d between conductors A and B is,

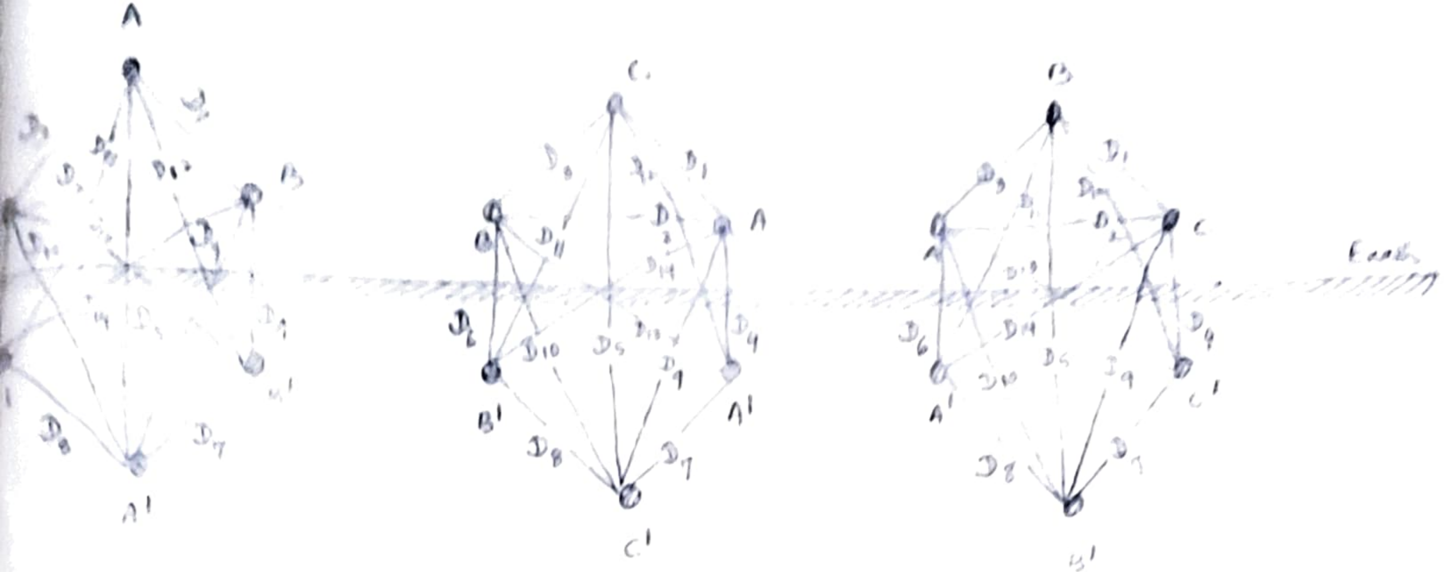
$$V_{AB} = V_A - V_B = \frac{2q}{2\pi\epsilon_0} \log_e \frac{2hD}{r\sqrt{4h^2+D^2}} = \frac{q}{\pi\epsilon_0} \log_e \frac{2hD}{r\sqrt{4h^2+D^2}}$$

∴ capacitance between conductors A and B is,

$$C_{AB} = \frac{q}{V_{AB}} = \frac{\pi\epsilon_0}{\log_e \frac{2hD}{r\sqrt{4h^2+D^2}}} \; F/m.$$

ffect of earth on the capacitance of 3 ϕ line :

→ Let us consider a 3 ϕ line A B C carry the changes q_A, q_B and q_C respectively. Let the line is transposed.



Position-1 Position-2 Position-3

→ Potential of conductor "A" in Position-1 is,

$$V_{A1} = \int_{r}^{\infty} \frac{q_A}{2\pi\epsilon_0 x} \cdot dx + \int_{D_5}^{\infty} \frac{-q_A}{2\pi\epsilon_0 x} dx + \int_{D_1}^{\infty} \frac{q_B}{2\pi\epsilon_0 x} \cdot dx + \int_{D_{12}}^{\infty} \frac{-q_B}{2\pi\epsilon_0 x} dx$$

$$+ \int_{D_3}^{\infty} \frac{q_C}{2\pi\epsilon_0 x} \cdot dx + \int_{D_{11}}^{\infty} \frac{-q_C}{2\pi\epsilon_0 x} \cdot dx$$

$$= \frac{1}{2\pi\epsilon_0} \left[q_A \log_e \frac{D_5}{r} + q_B \log_e \frac{D_{12}}{D_1} + q_C \log_e \frac{D_{11}}{D_3} \right]$$

Potential of conductor 'A' in Position 2 is,

$$V_{A2} = \int_{r}^{\infty} \frac{q_A}{2\pi\epsilon_0 x} \cdot dx + \int_{D_4}^{\infty} \frac{-q_A}{2\pi\epsilon_0 x} \cdot dx + \int_{D_2}^{\infty} \frac{q_B}{2\pi\epsilon_0 x} \cdot dx + \int_{D_{14}}^{\infty} \frac{-q_B}{2\pi\epsilon_0 x} \cdot dx$$

$$+ \int_{D_1}^{\infty} \frac{q_c}{2\pi\epsilon_0 x} \cdot dx + \int_{D_9}^{\infty} \frac{-q_c}{2\pi\epsilon_0 x} \cdot dx$$

$$= \frac{1}{2\pi\epsilon_0} \left[q_A \log_e \frac{D_4}{r} + q_B \log_e \frac{D_{14}}{D_2} + q_c \log_e \frac{D_9}{D_1} \right]$$

Potential of conductor 'A' in Position-3 is,

$$V_{A3} = \int_{r}^{\infty} \frac{q_A}{2\pi\epsilon_0 x} \cdot dx + \int_{D_6}^{\infty} \frac{-q_A}{2\pi\epsilon_0 x} \cdot dx + \int_{D_3}^{\infty} \frac{q_B}{2\pi\epsilon_0 x} \cdot dx + \int_{D_{10}}^{\infty} \frac{-q_B}{2\pi\epsilon_0 x} \cdot dx$$

$$+ \int_{D_2}^{\infty} \frac{q_c}{2\pi\epsilon_0 x} \cdot dx + \int_{D_{13}}^{\infty} \frac{-q_c}{2\pi\epsilon_0 x} \cdot dx$$

$$= \frac{1}{2\pi\epsilon_0} \left[q_A \log_e \frac{D_6}{r} + q_B \log_e \frac{D_{10}}{D_3} + q_c \log_e \frac{D_{13}}{D_2} \right]$$

∴ The average potential of conductor 'A' is,

$$V_A = \frac{V_{A1} + V_{A2} + V_{A3}}{3}$$

$$= \frac{1}{6\pi\epsilon_0} \left[q_A \log_e \frac{D_4 D_5 D_6}{r^3} + q_B \log_e \frac{D_{10} D_{12} D_{14}}{D_1 D_2 D_3} + q_c \log_e \frac{D_9 D_{11} D_{13}}{D_1 D_2 D_3} \right]$$

From figure $D_9 = D_{12}$

$D_{13} = D_{14}$

$D_{10} = D_{11}$

$$\therefore V_A = \frac{1}{6\pi\epsilon_0}\left[q_A \log_e \frac{D_4 D_5 D_6}{r^3} + q_B \log_e \frac{D_{11} D_9 D_{13}}{D_1 D_2 D_3} + q_C \log_e \frac{D_9 D_{11} D_{13}}{D_1 D_2 D_3}\right]$$

$$\Rightarrow V_A = \frac{1}{6\pi\epsilon_0}\left[q_A \log_e \frac{D_4 D_5 D_6}{r^3} + (q_B + q_C) \log_e \frac{D_{11} D_9 D_{13}}{D_1 D_2 D_3}\right]$$

$$V_A = \frac{1}{6\pi\epsilon_0}\left[q_A \log_e \frac{D_4 D_5 D_6}{r^3} - q_A \log_e \frac{D_{11} D_9 D_{13}}{D_1 D_2 D_3}\right]$$

$$V_A = \frac{1}{6\pi\epsilon_0} \cdot q_A \log_e \frac{D_4 D_5 D_6}{r^3} \times \frac{D_1 D_2 D_3}{D_{11} D_9 D_{13}}$$

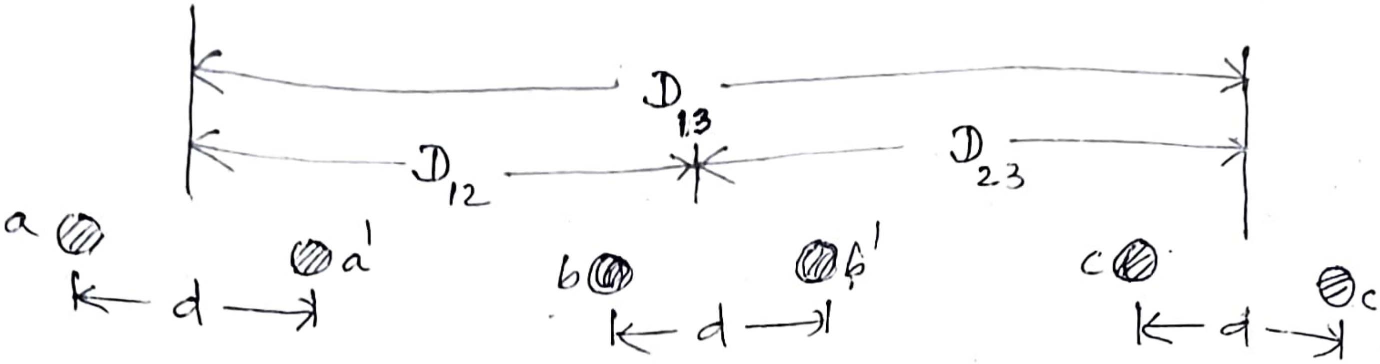
$$V_A = \frac{1}{2\pi\epsilon_0} q_A \cdot \log_e \frac{(D_4 D_5 D_6)^{\frac{1}{3}}}{r} \left(\frac{D_1 D_2 D_3}{D_{11} D_9 D_{13}}\right)^{\frac{1}{3}}$$

$\therefore$ Capacitance of conductor 'A' is

$$C_A = \frac{q_A}{V_A} = \frac{2\pi\epsilon_0}{\log_e \frac{(D_4 D_5 D_6)^{\frac{1}{3}}}{r} \left(\frac{D_1 D_2 D_3}{D_{11} D_9 D_{13}}\right)^{\frac{1}{3}}} \text{ F/m.}$$

Capacitance of Bundled Conductors (3-φ

Consider a 3-φ line as shown in fig

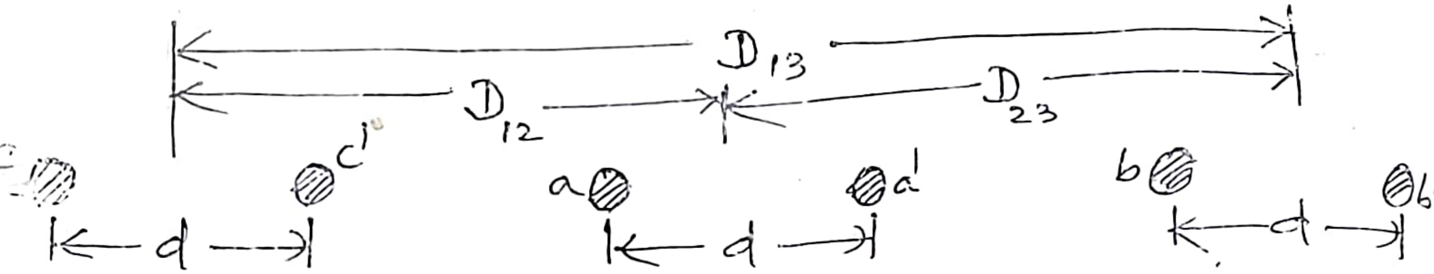


Position-①

Potential of conductor "a" in position-1.

$$V_{a1} = \frac{q_a/2}{2\pi\epsilon_0}\ln\left(\frac{\infty}{r}\right) + \frac{q_a/2}{2\pi\epsilon_0}\ln\left(\frac{\infty}{d}\right) + 2\cdot\frac{q_b/2}{2\pi\epsilon_0}\ln\left(\frac{\infty}{D_{12}}\right) + 2\cdot\frac{q_c/2}{2\pi\epsilon_0}\ln\left(\frac{\infty}{D_{13}}\right)$$

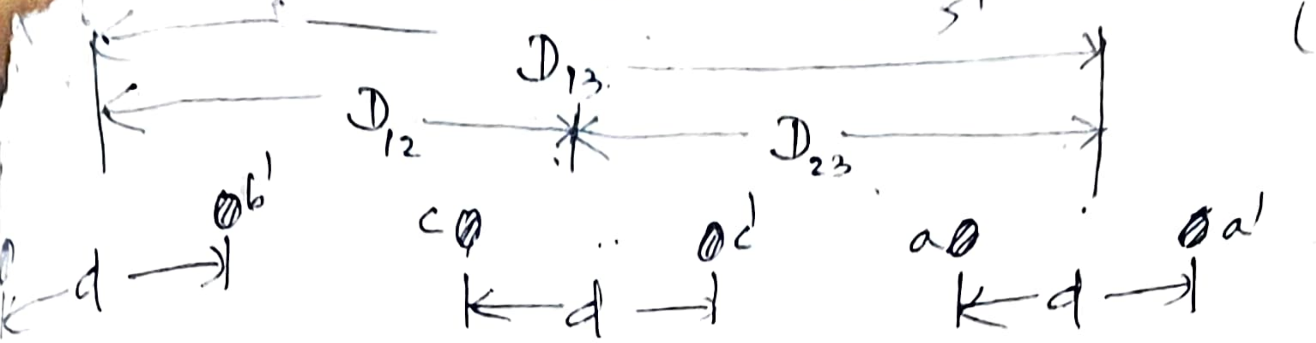
$$= \frac{q_a}{2\pi\epsilon_0}\ln\left(\frac{\infty}{\sqrt{dr}}\right) + \frac{q_b}{2\pi\epsilon_0}\ln\left(\frac{\infty}{D_{12}}\right) + \frac{q_c}{2\pi\epsilon_0}\ln\left(\frac{\infty}{D_{13}}\right)$$



Position-②

Potential of conductor 'a' at position-2

$$V_{a2} = \frac{q_a}{2\pi\epsilon_0}\ln\frac{\infty}{\sqrt{dr}} + \frac{q_c}{2\pi\epsilon_0}\ln\left(\frac{\infty}{D_{12}}\right) + \frac{q_B}{2\pi\epsilon_0}\ln\left(\frac{\infty}{D_{2}}\right.$$



Position-③

otential of conductor 'a' at position-③,

$$V_{a3} = \frac{q_a}{2\pi\epsilon_0}\ln\left(\frac{\infty}{\sqrt{dr}}\right) + \frac{q_b}{2\pi\epsilon_0}\ln\left(\frac{\infty}{D_{13}}\right) + \frac{q_c}{2\pi\epsilon_0}\ln\left(\frac{\infty}{D_{23}}\right)$$

$$V_a = \frac{V_{a1}+V_{a2}+V_{a3}}{3}$$

$$\Rightarrow V_a = \frac{1}{3}\left[\frac{3q_a}{2\pi\epsilon_0}\ln\left(\frac{\infty}{\sqrt{dr}}\right) + \frac{q_b}{2\pi\epsilon_0}\ln\left(\frac{\infty^3}{D_{12}D_{23}D_{13}}\right) + \frac{q_c}{2\pi\epsilon_0}\ln\left(\frac{\infty^3}{D_{12}D_{23}D_{13}}\right)\right]$$

$$V_a = \frac{q_a}{2\pi\epsilon_0}\ln\frac{\infty}{\sqrt{dr}} + \frac{1}{3}\frac{1}{2\pi\epsilon_0}\ln\frac{\infty^3}{D_{12}D_{23}D_{13}}(q_b+q_c)$$

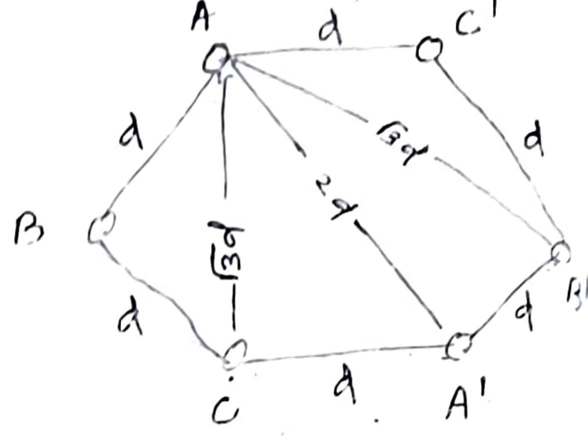
$$V_a = \frac{q_a}{2\pi\epsilon_0}\ln\frac{\infty}{\sqrt{dr}} - \frac{q_a}{2\pi\epsilon_0}\ln\frac{\infty}{\sqrt[3]{D_{12}D_{23}D_{13}}} \quad \left\{\because q_b+q_c=-\right.$$

$$V_a = \frac{q_a}{2\pi\epsilon_0}\left[\ln\frac{\sqrt[3]{D_{12}D_{23}D_{13}}}{\sqrt{dr}}\right] = \frac{q_a}{2\pi\epsilon_0}\ln\frac{D_{eq}}{\sqrt{dr}}.$$

$$\therefore \text{Capacitance, } C_a = \frac{q_a}{V_a} = \frac{2\pi\epsilon_0}{\ln\left(\frac{D_{eq}}{\;}\right)} \text{ F/m.}$$

Capacitance of double circuit 3-ϕ line

(1) Symmetrical spaced line:



→ Let us consider a 3-ϕ line double ckt as shown in fig.

→ Let the charge over conductors A, B and C be q_A, q_B and q_C coulombs per metre length. Then charge over conductors A', B' and C' will be q_A, q_B and q_C coulombs per metre l

$$q_A + q_B + q_C = 0.$$

→ Potential of conducter A w.r.t neutral infinite pl

$$V_A = \frac{1}{2\pi\epsilon_0}\left[q_A \ln\frac{\infty}{r} + q_B \ln\frac{\infty}{d} + q_C \ln\frac{\infty}{\sqrt{3}d} + q_A \ln\frac{\infty}{2d} + q_B \ln\frac{\infty}{\sqrt{3}} + q_C \ln\frac{\infty}{d}\right]$$

$$= \frac{1}{2\pi\epsilon_0}\left[q_A \ln\frac{\infty}{2dr} + q_B \ln\frac{\infty}{\sqrt{3}d^2} + q_C \ln\frac{\infty}{\sqrt{3}d^2}\right]$$

$$= \frac{1}{2\pi\epsilon_0}\left[q_A \ln\frac{\infty}{2dr} + \ln\frac{\infty}{\sqrt{3}d^2}(q_B + q_C)\right] \quad \left\{\because q_B + q_C = \right.$$

$$= \frac{1}{2\pi\epsilon_0}\left[q_A \ln\frac{\infty}{2dr} - q_A \ln\frac{\infty}{\sqrt{3}d^2}\right]$$

$$= \frac{1}{2\pi\epsilon_0} q_A \ln\frac{\sqrt{3}d^2}{2dr} = \frac{1}{2\pi\epsilon_0} q_A \ln\frac{\sqrt{3}d}{2r}$$

∴ Capacitance of conducts A,

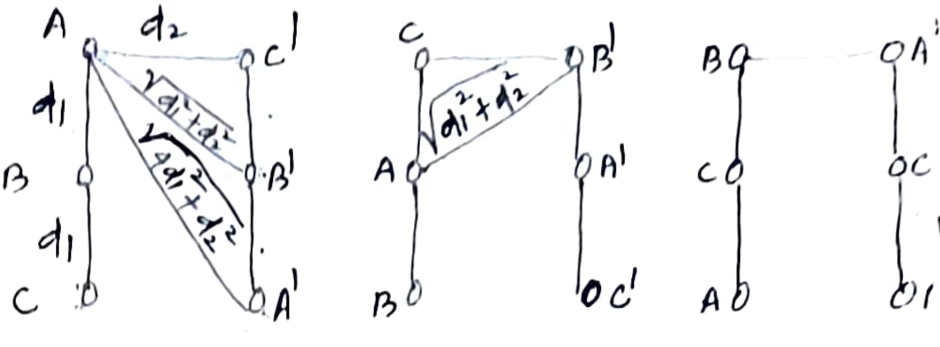
$$C_A = \frac{q_A}{V_A} = \frac{2\pi\epsilon_0}{\ln\frac{\sqrt{3}d}{2r}} \text{ F/m}.$$

But A and A' are conductors connected in parallel forming one p

∴ capacitance per phase, $C = C_A + C_A = 2C_A = \frac{4\pi\epsilon_0}{\ln\sqrt{3}d/2r}$ F/m

ii) Unsymmetrical Spacing (Transposed lines) :-

→ Let us consider a 3-ϕ line double ckt connected in parallel conductors A, B, C forming one ckt and conductors A', B', C' forming the other.



Position-1 Position-2 Position-

→ Figure shows the conductors unsymmetrically spaced an transposed.

→ Potential of conductor 'A' w.r.t infinite neutral plane in position-1 is,

$$V_{A1} = \frac{1}{2\pi\epsilon_0}\left[q_A \ln\frac{\infty}{r} + q_B \ln\frac{\infty}{d_1} + q_C \ln\frac{\infty}{2d_1} + q_A \ln\frac{\infty}{\sqrt{4d_1^2+d_2^2}} + q_B \ln\frac{\infty}{\sqrt{d_1^2+d_2^2}} + q_C \ln\frac{\infty}{d_2}\right]$$

$$= \frac{1}{2\pi\epsilon_0}\left[q_A \ln\infty - q_A \ln r + q_B \ln\infty - q_B \ln d_1 + q_C \ln\infty - q_C \ln 2d_1 + q_A \ln\infty - q_A \ln\sqrt{4d_1^2+d_2^2} + q_B \ln\infty - q_B \ln\sqrt{d_1^2+d_2^2} + q_C \ln\infty - q_C \ln d_2\right]$$

$$= \frac{1}{2\pi\epsilon_0}\left[\ln\infty\,(q_A+q_B+q_C) + q_A \ln\frac{1}{r\sqrt{4d_1^2+d_2^2}} + q_B \ln\frac{1}{d_1\sqrt{d_1^2+d_2^2}} + q_C \ln\frac{1}{2d_1d_2}\right]$$

$$= \frac{1}{2\pi\epsilon_0}\left[q_A \ln\frac{1}{r\sqrt{4d_1^2+d_2^2}} + q_B \ln\frac{1}{d_1\sqrt{d_1^2+d_2^2}} + q_C \ln\frac{1}{2d_1d}\right.$$

Similarly, Potential of conductor A w.r.t neutr plane in position-2 is,

$$V_{A2} = \frac{1}{2\pi\epsilon_0}\left[q_A \ln\frac{1}{r d_2} + q_B \ln\frac{1}{d_1\sqrt{d_1^2+d_2^2}} + q_c \ln\frac{1}{d_1\sqrt{d_1^2+d_2^2}}\right]$$

$$V_{A3} = \frac{1}{2\pi\epsilon_0}\left[q_A \ln\frac{1}{r\sqrt{4d_1^2+d_2^2}} + q_B \ln\frac{1}{2d_1 d_2} + q_c \ln\frac{1}{d_1\sqrt{d_1^2+d}}\right.$$

$$\therefore V_A = \frac{V_{A1}+V_{A2}+V_{A3}}{3}$$

$$= \frac{q_A}{2\pi\epsilon_0} \ln 2^{\frac{1}{3}} \cdot \frac{d_1}{r}\left(\frac{d_1^2+d_2^2}{4d_1^2+d_2^2}\right)^{1/3}$$

Capacitance of conductor 'A' is,

$$C_A = \frac{q_A}{V_A} = \frac{2\pi\epsilon_0}{\ln 2^{\frac{1}{3}} \cdot \frac{d_1}{r}\left(\frac{d_1^2+d_2^2}{4d_1^2+d_2^2}\right)^{\frac{1}{3}}} \text{ F/m.}$$

Since conductors are electrically in Parallel, then capacitance, per phase

$$C = C_A + C_A = 2C_A$$

$$\Rightarrow C = \frac{4\pi\epsilon_0}{\ln 2^{\frac{1}{3}} \cdot \frac{d_1}{r}\left(\frac{d_1^2+d_2^2}{4d_1^2+d_2^2}\right)^{1/3}} \text{ F/m.}$$

EPTD

Classification of Overhead lines :-

→ The overhead lines are classified depending upon the manner in which capacitance is taken into account. They are classified as short, medium and long transmission lines.

Short transmission lines :-

Transmission lines having length lesser than 80 Km and line voltage in comparatively low (< 20KV) are considered as short transmission line. Due to small length and low voltage, the capacitance effects are small. Hence the performance of short transmission lines depend upon resistance and inductance of the line.

Medium transmission Lines :-

Transmission lines having length between 80 Km and 200 Km and line voltage between 20 KV and 100 KV are considered as Medium transmission lines. Due to sufficient length and voltage of the line, the capacitance effects are taken into account.

Long transmission lines :-

Transmission lines having length above 200 km and line voltage above 100 KV are considered as Long transmission lines. In these lines impedance and admittance are uniformly distributed (not lumped) and rigorous methods are employed for solution.

Voltage regulation: When a transmission line car current then there is a volt drop in the line due to resist and inductance.

As a result the receiving end voltage of the lin is less than the sending end voltage. This voltage drop in the line expressed as a percentage of receiving end voltage and is called voltage regulation.

The difference in voltage at the receiving end of a transmission lin between conditions of no load and full load is called voltage regulation is expressed as a percentage of receiving end end voltage of the line.

Let V_R = receiving end voltage of the line.
V_S = sending end voltage of the line

$$\therefore \% \text{ of voltage regulation} = \frac{V_S - V_R}{V_R} \times 100$$

The lower the voltage regulation, better it is. Because low voltage regulation means little variation in receiving end voltage due to variation in load current.

Transmission efficiency:- When load is supplied there are line losses due to ohmic resistance of line conductors, so power obtained at the receiving end of transmission line is less than the sending end power.

→ Efficiency of a transmission line is defined as the ratio of Power delivered at the receiving end to the Power sent from sending end. It is represented by η_T.

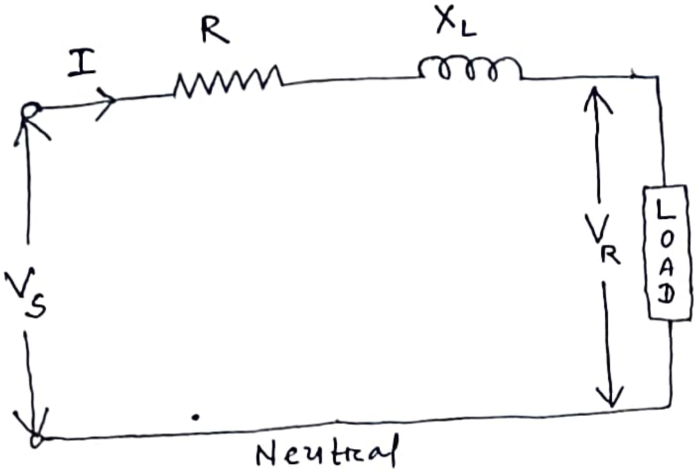
$$\therefore \eta_T = \frac{\text{receiving end Power}}{\text{sending end power}} \times 100$$

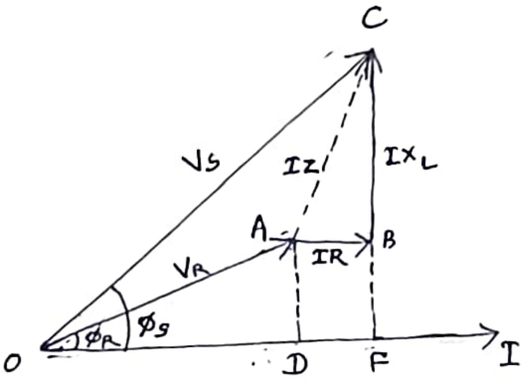
$$= \frac{V_R I_R \cos\phi_R}{V_s I_s \cos\phi_s} \times 100$$

Short transmission lines :-

→ In short transmission line the effect of Shunt capacitance and shunt conductance are neglected. and only resistance & inductive reactance are taken into account. Let resistance & inductance are concentrated (Lumped) instead of being distributed.

→ Let us consider a 3-ϕ short transmission line as shown in figure, the load is applied in one phase.





Let R = resistance per phase

X_L = inductive reactance per phase

I = Current in one conductor

V_R = receiving end voltage to neutral

V_S = sending end voltage to neutral.

$V_R = V_S - I(R + jX_L) = V_S - IZ$

Where IZ = voltage drop along the line.

From fig, $OA = V_R$, $OC = V_S$
$AB = IR$ $OD = V_R \cos\phi_R$
$BC = IX_L$ $AD = BF = V_R \sin\phi_R$
$AC = IZ$

(i) $OC = \sqrt{OF^2 + FC^2} = \sqrt{(OD+DF)^2 + (FB+BC)^2} = \sqrt{(V_R\cos\phi_R + IR)^2 + (V_R\sin\phi$

(ii) ϕ_S = sending end Phase angle $= \tan^{-1}\frac{FC}{OF} = \tan^{-1}\left(\frac{BF+BC}{OD+DF}\right)$

$$= \tan^{-1}\left(\frac{V_R\sin\phi_R + IX_L}{V_R\cos\phi_R + IR}\right) \longrightarrow (2)$$

(iii) Sending end power factor $= \cos\phi_S = \frac{OF}{OC} = \frac{V_R\cos\phi_R + IR}{V_S} \rightarrow$ ③

(iv) Percentage voltage regulation $= \frac{V_S - V_R}{V_R} \times 100 \longrightarrow$ ④

v) Power delivered to load $= V_R I \cos\phi_R$

Line losses $= I^2 R$

Power sent out $= V_R I \cos\phi_R + I^2 R$

$\therefore$ % Transmission efficiency $= \frac{\text{Power delivered}}{\text{Power sent out}} \times 100$

$$= \frac{V_R I \cos\phi_R}{V_R I \cos\phi_R + I^2 R} \times 100 \longrightarrow$$

per unit. V.R for short trans. line

$$= \frac{I_R R}{V_R}\cos\phi_R + \frac{I_R X}{V_R}\sin\phi_R$$

$$= v_r \cos\phi_r + v_x \sin\phi_r$$

) From Equation ① $V_S = \sqrt{(V_R\cos\phi_R + IR)^2 + (V_R\sin\phi_R + IX_L)^2}$

$$\Rightarrow V_S = \sqrt{V_R^2\cos^2\phi_R + 2V_R\cos\phi_R\cdot IR + I^2R^2 + V_R^2\sin^2\phi_R + 2V_R\sin\phi_R\cdot IX_L + I^2X_L^2}$$

$$\Rightarrow V_S = \sqrt{V_R^2 + 2V_R\cdot IR\cdot\cos\phi_R + 2V_R\cdot IX_L\cdot\sin\phi_R + I^2(R^2+X_L^2)}$$

$$\Rightarrow V_S = \sqrt{V_R^2\left[1 + \frac{2IR}{V_R}\cdot\cos\phi_R + \frac{2IX_L}{V_R}\cdot\sin\phi_R + \frac{I^2}{V_R^2}(R^2+X_L^2)\right]}$$

$$\Rightarrow V_S = V_R\sqrt{1 + \frac{2IR}{V_R}\cos\phi_R + \frac{2IX_L}{V_R}\sin\phi_R}$$

The term $\frac{I^2}{V_R^2}(R^2+X_L^2)$ has a small value. So it can be neglected

$$V_S = V_R\left[1 + \left(\frac{2IR\cos\phi_R}{V_R} + \frac{2IX_L\sin\phi_R}{V_R}\right)\right]^{\frac{1}{2}}$$

Note

$(1+x)^{\frac{1}{2}} = 1 + \frac{1}{2}x + \frac{\frac{1}{2}(\frac{1}{2}-1)}{2}x^2 + \dots$

$= 1 + \frac{x}{2} + \frac{1}{8}x^2 + \dots$

$= 1 + \frac{x}{2}\cdot$ neglecting $\frac{1}{8}x^2$ term

$$\Rightarrow V_S = V_R\left[1 + \frac{\frac{2IR\cos\phi_R}{V_R} + \frac{2\cdot IX_L\sin\phi_R}{V_R}}{2}\right]$$

$$\Rightarrow \boxed{V_S = V_R + IR\cos\phi_R + IX_L\sin\phi_R} \longrightarrow ⑥$$

(vii) Voltage regulation $= \frac{V_S - V_R}{V_R}\times 100$

per unit V.R $= \frac{IR\cos\phi_R + IX_L\sin\phi_R}{V_R}\times 100.$ $\longrightarrow$ ⑦

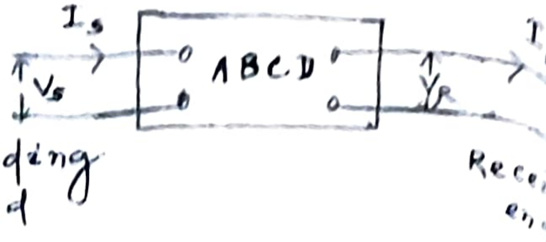
Equation ⑦ is for Lagging load.

But for Leading load, voltage regulation $= \frac{IR\cos\phi_R - IX_L\sin\phi_R}{V_R}\times 100$

For unit P.f, voltage regulation $= \frac{IR}{V_R}\times 100$ $\{\because \cos\phi = 1$ & $\sin\phi = 0\}$

(viii) → A 3-φ transmission line can be represented by a ckt with two input terminals and two output terminals.

→ In this ckt the voltage & current on the receiving end and sending end are related by the following equations:



$$V_S = AV_R + BI_R$$
$$I_S = C.V_R + DI_R$$

Where A, B, C and D are called the constants of th network.

→ In short transmission line $V_S = V_R + I_R Z$
and $I_S = I_R$

→ Comparing these eqs we get, $A = 1$, $B = \cancel{I_R} Z$
and $C = 0$, $D = 1$

Problem: An overhead 3-φ transmission line delivers 5000 KW at 22 KV at 0.8 Pf lagging. The resistance and reactance of each conductor is 4Ω and 6Ω respectively. Determine (i) sending end voltage (ii) Percentage regulation (iii) transmission efficiency.

Solution:- Let it is star connected system.

load P.f = $\cos\phi_R = 0.8$ (lag)

Receiving end voltage/phase, $V_R = \frac{22 \times 10^3}{\sqrt{3}} = 12700$ V.

Impedance per phase, $Z = R + jX_L = 4 + j6$

Line current, $I = \frac{5000 \times 10^3}{3 \times 12700 \times 0.8} = 164$ A

(i) Sending end voltage/phase is,

$$V_s = \sqrt{(V_R \cos\phi_R + IR)^2 + (V_R \sin\phi_R + IX_L)^2}$$

$$= \sqrt{(12700 \times 0.8 + 164 \times 4)^2 + (12700 \times 0.6 + 164 \times 6)^2}$$

$$= 13820.8 \text{ volt.}$$

Line value of $V_s = \sqrt{3} \times 13820.8 = 23938$ V

(ii) % regulation $= \frac{V_s - V_R}{V_R} \times 100 = \frac{13820.8 - 12700}{12700} \times 100$

$= 8.825$ %

(iii) line losses $= 3I^2R = 3(164)^2 \times 4 = 322752$ watt.

∴ Transmission efficiency $= \frac{5000}{5000 + 322.752} \times 100$

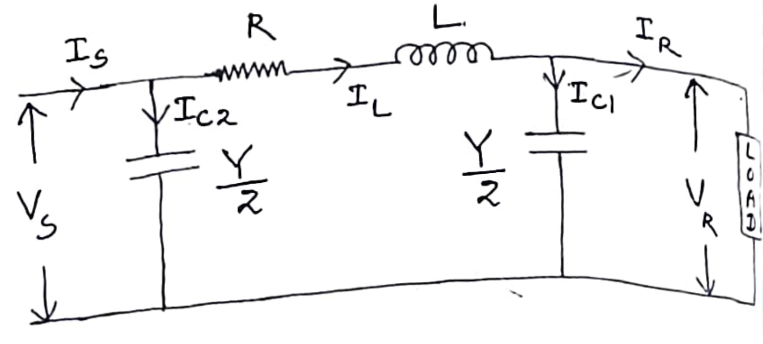
$= 93.94$ %.

Medium transmission lines :-

→ In Medium transmission lines, the line capacitance must be taken into consideration. The capacitance is distributed over the entire length of the line. In order to make the calculations simple, the line capacitance is assumed to be concentrated (or lumped) in the form of capacitors shunted across the line at one or more points.

→ The most commonly used methods for the solution of medium transmission lines are,
(i) Nominal π method. (ii) Nominal T method.

→ **Nominal π method :-** In this method, capacitance of each conductor (i.e line to neutral) is divided into two parts. One half being concentrated (lumped) at sending end and the other half concentrated at receiving end as shown in fig.



et I_R = Load current per phase

R = resistance per phase.

X_L = inductive reactance per phase.

C = capacitance per phase.

Y = jωC = shunt admittance.

Z = R + jX_L = impedance per phase.

V_S = sending end voltage per phase.
V_R = receiving end voltage.

$$I_{C1} = \frac{V_R Y}{2}$$

$$I_L = I_R + I_{C1} = I_R + \frac{V_R Y}{2}$$

$$\therefore V_S = \left(I_R + \frac{V_R Y}{2}\right) Z + V_R$$

$$\Rightarrow V_S = \left(\frac{YZ}{2} + 1\right) V_R + Z I_R \longrightarrow ①$$

$$I_{C2} = \frac{V_S Y}{2}$$

$$I_S = I_L + I_{C2}$$

$$= I_R + \frac{V_R Y}{2} + \frac{V_S Y}{2}$$

$$= I_R + \frac{V_R Y}{2} + \left[\left(\frac{YZ}{2} + 1\right) V_R + Z I_R\right] \frac{Y}{2}$$

$$= I_R + \frac{V_R Y}{2} + \left(\frac{YZ}{2} + 1\right) \frac{V_R Y}{2} + I_R \frac{YZ}{2}$$

$$= V_R Y\left(\frac{1}{2} + \frac{YZ}{4} + \frac{1}{2}\right) + \left(\frac{YZ}{2} + 1\right) I_R$$

$$= V_R Y\left(1 + \frac{YZ}{4}\right) + \left(\frac{YZ}{2} + 1\right) I_R \longrightarrow ②$$

But $V_S = A V_R + B I_R \longrightarrow ③$

and $I_S = C V_R + D I_R \longrightarrow ④$

Comparing these equations, we get

$$A = 1 + \frac{YZ}{2}, \quad B = Z$$

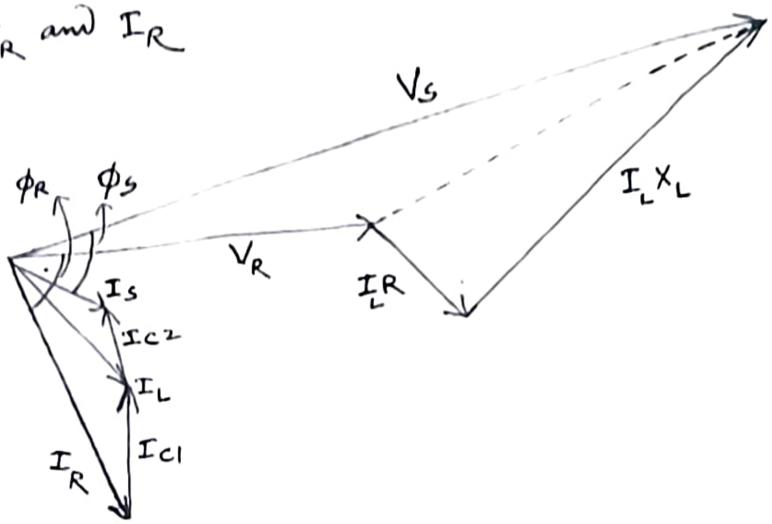
$$C = Y\left(1 + \frac{YZ}{4}\right), \quad D = 1 + \frac{YZ}{2}$$

Where A, B, C and D are called generalized circuit constants of the transmission lines.

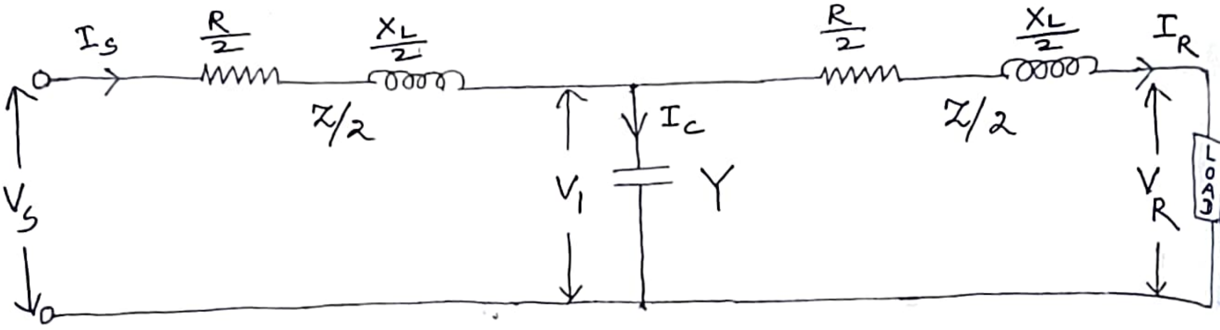
The Phasor diagram shown in figure.

ϕ_s = angle between V_s and I_s

ϕ_R = angle between V_R and I_R



Nominal T Method :- In this method, the whole line capacitance is assumed to be concentrated at the middle point of the line and half the line resistance and reactance are concentrated on its either side as shown in figure.



$$Z = R + jX_L$$

$$\therefore \frac{Z}{2} = \frac{R + jX_L}{2}$$

I_R = load current

R = resistance per phase

X_L = inductive reactance per phase.

C = capacitance per phase.

Y = jwC = shunt admittance.

V_1 = voltage across capacitance.

$$V_1 = V_R + I_R\left(\frac{Z}{2}\right)$$

$$I_C = V_1 Y = Y\left(V_R + I_R\cdot\frac{Z}{2}\right)$$

$$I_S = I_R + I_C$$

$$\Rightarrow I_S = I_R + YV_R + I_R\cdot\frac{YZ}{2}$$

$$\Rightarrow I_S = YV_R + \left(1+\frac{YZ}{2}\right)I_R \longrightarrow ①$$

$$V_S = V_1 + I_S\cdot\frac{Z}{2}$$

$$\Rightarrow V_S = V_1 + \left[YV_R + \left(1+\frac{YZ}{2}\right)I_R\right]\frac{Z}{2}$$

$$\Rightarrow V_S = V_R + I_R\cdot\frac{Z}{2} + \left[V_R\frac{YZ}{2} + I_R\cdot\frac{Z}{2} + I_R\cdot\frac{YZ^2}{4}\right]$$

$$\Rightarrow V_S = V_R\left(1+\frac{YZ}{2}\right) + I_R\left(Z+\frac{YZ^2}{4}\right) \longrightarrow ②$$

But we know, $V_S = AV_R + BI_R \longrightarrow ③$

and $I_S = CV_R + DI_R \longrightarrow ④$

Comparing these equations, we get

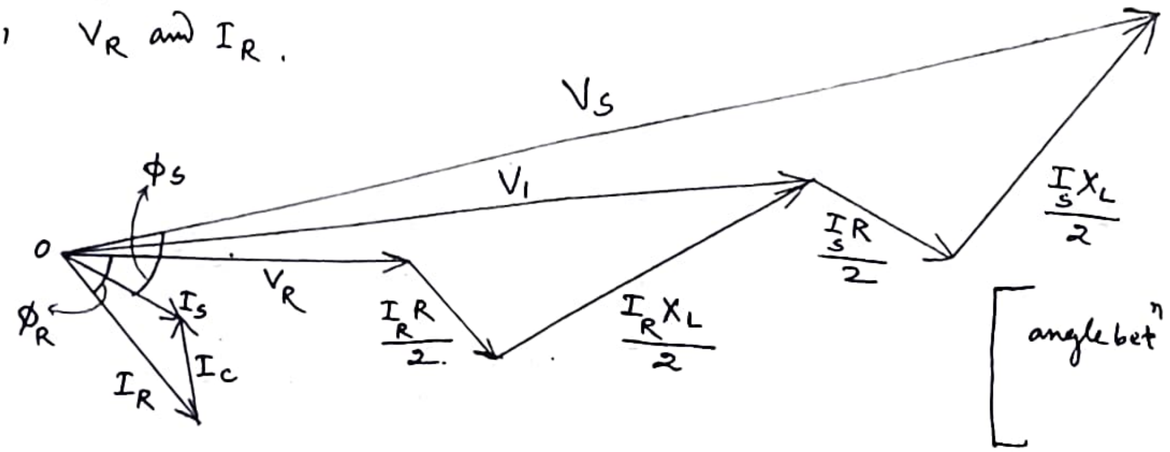
$A = 1+\frac{YZ}{2}$, $B = Z+\frac{YZ^2}{4} = Z\left(1+\frac{YZ}{4}\right)$

$C = Y$, $D = 1+\frac{YZ}{2}$

The phasor diagram is shown in figure.

ϕ_S = angle betn V_S and I_S

ϕ_R = " " V_R and I_R.



[angle betn V_1

Problem:- Find the sending end voltage and voltage regulation of 250 km, 3-φ, 50 Hz transmission line delivering 25 MVA at 0.8 p.f lagging to a balanced load at 132 KV. The line has series impedance of 27.5 + j97.4 ohms and shunt admittance 7.38×10^{-7} ℧. Neglect leakage reactance. (use nominal π method)

Solution:- Line voltage at receiving end = 132×10^3 V.

Phase voltage at receiving end $V_R = \frac{132 \times 10^3}{\sqrt{3}}$

$= 76212.47$

Receiving end current $I_R = \frac{25 \times 10^6}{\sqrt{3} \times 132 \times 1000} = 109.35$ A

Taking phase voltage at receiving-end V_R as reference.

$V_R = 76210 + j0$.

$I_R = 109.35(0.8 - j0.6) = 87.48 - j65.6$ A

Charging current at receiving end $I_{C1} = \frac{Y}{2} V_R$

$= \frac{1}{2}(0 + j7.38 \times 10^{-4}) \times (76210 + j0)$

$= 0 + j28.12$ A

Line current $I_L = I_R + I_{C1} = 87.48 - j65.6 + 0 + j28.12$

$= 87.48 - j37.48$ A

Sending end voltage, $V_S = V_R + I_L Z = (76210 + j0) + (87.48 - j37.48) \times (27.5 + j97.4)$

$= 82615 \angle 5.27$ V.

Line voltage at sending end $V_{SL} = \sqrt{3} \times 82615$

$= 143093$ V.

We Know that $V_S = AV_R + BI_R$

At no load, $V_S = AV_R + 0$

$$\Rightarrow V_R = \frac{V_S}{A} = \frac{V_S}{1+\frac{YZ}{2}} \quad \left\{ \because A = 1 + \frac{YZ}{2} \text{ for Medium Transmission line} \right.$$

so line voltage at receiving-end on no load is

$$\dot{V}_R' = \frac{V_{SL}}{1+\frac{YZ}{2}} = \frac{143093}{1+\frac{(j7.38\times10^{-4})(27.5+j97.4)}{2}}$$

$$= \frac{143093}{0.964} \quad \text{(Consider only magnitude)}$$

$= 148436.722$ Volt.

$\therefore$ Voltage regulation $= \dfrac{V_R' - V_R}{V_R} \times 100$

$$= \frac{148436.722 - 132\times10^3}{132\times10^3} \times 100$$

$= 12.4\,\%$

Exercise Problem 4.7
BOOK C.L WADHWA

② A 3-ϕ, 50 Hz transmission line has resistance, inductance and capacitance per phase of 10 Ω, 0.1 H and 0.9 μF respectively and delivers a load of 35 MW at 132 kV and 0.8 P.f lag. Determine efficiency and regulation of the line using (i) nominal-T and (ii) nominal π method.

(i) Nominal T Method:- (14)

Solution:- Resistance per phase $R = 10\,\Omega$

Inductive reactance per phase, $X_L = 2\pi f L = 314 \times 0.1$

$= 31.4\,\Omega$

Capacitive reactance per phase $X_c = \frac{1}{wC}$

" shunt admittance per phase, $Y = jwC = j2.826$

$\cos\phi = 0.8$, $\sin\phi = 0.6$, $Z = 10 + j31.4\,\Omega = 32.95\underline{\lvert 72.}$

$\frac{Z}{2} = \frac{R}{2} + j\frac{X_L}{2} = 5 + j15.7\,\Omega$

Voltage across Capacitance

Receiving end line voltage $= 132 \times 10^{3}$ V.

" " phase voltage $= V_R = \frac{132 \times 10^{3}}{\sqrt{3}}$

$= 76212.47$ V.

Receiving end current $I_R = \frac{P}{\sqrt{3}\, V_L \cos\phi} = \frac{35 \times 10^{6}}{\sqrt{3} \times 132 \times 10^{3} \times 0.8}$

$= 191.36$ A

$I_R = 191.36\underline{\lvert 36.869} = 153.08 - j114.81$

We know that, the transmission lime constants ar

$A = 1 + \frac{YZ}{2} = 1 + (j2.826 \times 10^{-4})(5 + j15.7)$

$= 0.9955 + j\,1.413 \times 10^{-3} = 0.995\underline{\lvert 0.08}$

$B = Z + \frac{YZ^{2}}{4} = 10 + j31.4 + \frac{(j2.826 \times 10^{-4})(-885.64 + j}{4}$

$= 10 + j31.4 - 0.04435 - j0.0625$

$= 9.955 + j31.337$

$C = Y = 0 + j2.826 \times 10^{-4}$

$D = A = 0.9955 + j\,1.413 \times 10^{-3}$

(15)

Sending end voltage $V_s = AV_R + BI_R$

$$= (0.4955 + j1.413\times10^{-3})(76212.47) +$$

$$(9.955 + j31.337)(153.08 - j114.81)$$

$$= 80991.22 + j3761.81$$

$$= 81078.53 \angle 2.66^\circ \text{ volts}$$

Sending end line voltage $= \sqrt{3} \times 81078.53$

$$= 140428.014 \text{ volts.}$$

At no load, $I_R = 0$.

we know, $V_S = AV_R + BI_R = AV_R + 0 = AV_R$.

$\Rightarrow V_R = \frac{V_s}{A}$

So no load receiving end voltage, $V_R' = \frac{140428.014}{A}$

$$= \frac{140428.014}{0.995} = 141133.6824 \text{ volts.}$$

$\therefore$ % regulation $= \frac{141133.6 - 132\times10^3}{132\times10^3}\times100$

$$= 6.9193\ \%$$

To determine η we evaluate transmission line losses as follows:

$$3\left[I_R^2\frac{R}{2} + I_S^2\times\frac{R}{2}\right] = \text{losses} \longrightarrow ①$$

We know $I_S = CV_R + DI_R = (j2.826\times10^{-4})(76212.47) +$

$$(0.9955 + j1.413\times10^{-3})(153.08 - j114.8)$$

$$= 152.55 - j92.53 = 178.41 \angle -31.23 \text{ A}$$

$\therefore$ losses = $3\left[36618.64 \times \frac{10}{2} + 31830.12 \times \frac{10}{2}\right]$

$= 1.0267 \times 10^{6}$ watt.

$\therefore$ % efficiency $= \frac{35 \times 10^{6}}{35 \times 10^{6} + 1.02 \times 10^{6}} \times 100$

$= 97.168$ %

The Long transmission Line :

→ In Long transmission line the line constants (resistance, inductance, capacitance and conductance) are uniformly distributed over the entire length of the line.

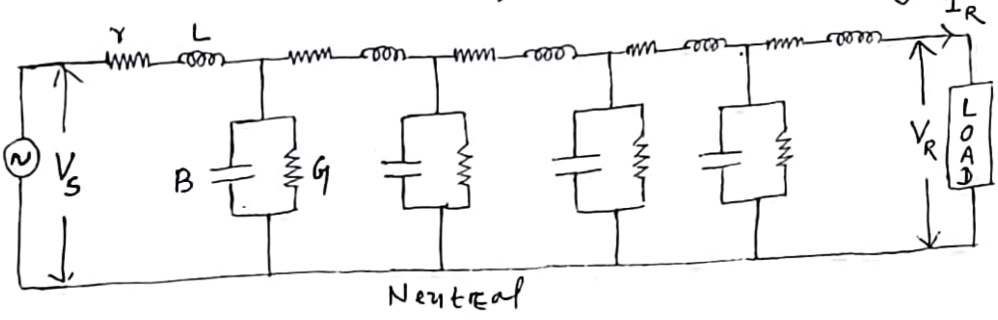
→ The Resistance (R) and inductive reactance (X) are the series elements.

→ The capacitive susceptance (B) and leakage conductance (G) are shunt elements. The leakage susceptance is due to the fact the capacitance exists between line and neutral. The leakage conductance is mainly due to flow of leakage currents over the surface of the insulators especially during bad weather. The leakage conductance takes into account the energy losses occurring through leakage over the insulators, or due to corona effect between the conductors.

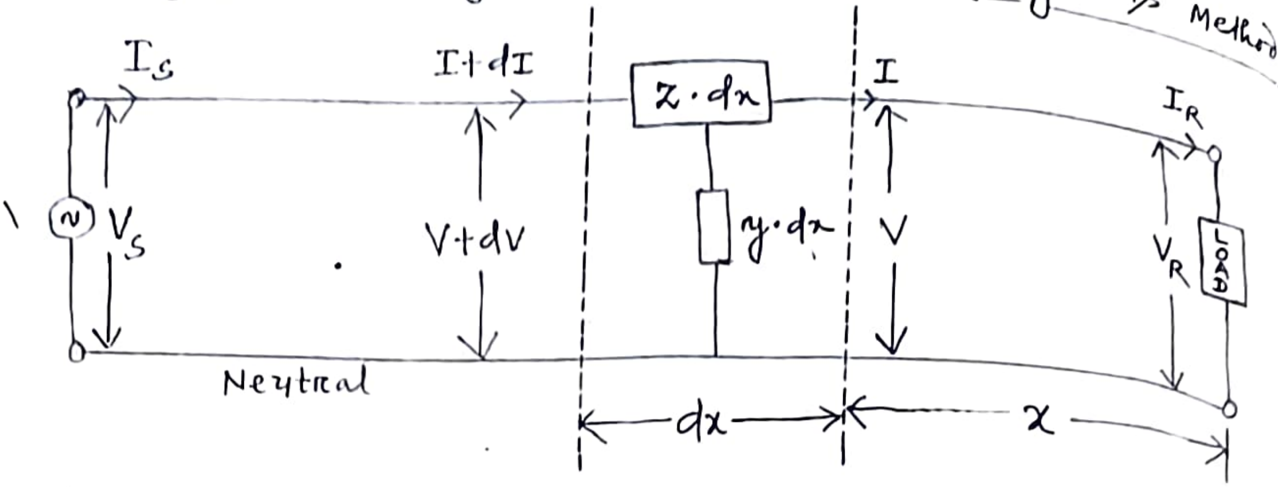
Shunt admittance $Y = \sqrt{G^2 + B^2}$

→ The leakage current through the shunt admittance is maximum at the sending end and decreases continuously as we move towards the receiving end.

→ The equivalent circuit of a 3-ϕ Long transmission line is shown in fig.



Analysis of Long transmission Lines (Rigorous Method)



→ Let us consider a transmission line represented on a single phase basis. For regorous solution we have to consider the line impedance and admitt uniformly distributed (and not lumped or concentra

→ Consider a small element of length dx situated at a distance 'x' from receiving e

Let r = resistance per unit length of line.
x = reactance per unit length of line.
b = susceptance per unit length of line.
g = conductance per unit length of line
$z = \sqrt{r^2 + x^2}$ = impedance per unit length of line
$y = \sqrt{g^2 + b^2}$ = admittance per unit length of lin

V = Votage per phase at the end of element towards re en
$V + dV$ = Voltage per phase at the end of element towards sending
V_R = Voltage per phase at receiving end.
V_S = Voltage per phase at sending end.
I_R = Current per phase at receiving end
I_S = Current per phase at sending end.
I = Current leaving the element dx
$I + dI$ = Current entering the element dx

Impedance of the element = $z . dx$

Shunt admittance of the element = $y . dx$

Rise in Voltage over the element in the direction of increasing x = dv

$\therefore$ Voltage = Current x impedance

$$\Rightarrow dV = I . z . dx$$

$$\Rightarrow \frac{dV}{dx} = Iz \longrightarrow ①$$

Difference of Current enterering the element and that of leaving the element = dI

$$\therefore dI = V . y dx$$

$$\Rightarrow \frac{dI}{dx} = Vy \longrightarrow ②$$

Differentiating Eqn (1) with respect to 'x' we get,

$$\frac{d^2V}{dx^2} = z . \frac{dI}{dx}$$

$$\Rightarrow \frac{d^2V}{dx^2} = z . Vy \longrightarrow ③ \left\{ \because \frac{dI}{dx} = Vy \right\}$$

The solution of eqn ③ is,

$$V = A_1 e^{x\sqrt{yz}} + A_2 e^{-x\sqrt{yz}} \longrightarrow ④$$

where A_1 and A_2 are constants.

Differentiating eqn ④ w.r.t 'x' we get,

$$\frac{dV}{dx} = \sqrt{yz} \left[A_1 e^{x\sqrt{yz}} - A_2 e^{-x\sqrt{yz}} \right]$$

From equation (1) we get,

$$I = \frac{1}{Z} \cdot \frac{dV}{dx}$$

$$\Rightarrow I = \frac{1}{Z} \cdot \sqrt{yZ} \left[A_1 e^{x\sqrt{yZ}} - A_2 e^{-x\sqrt{yZ}} \right]$$

$$\Rightarrow I = \sqrt{y/Z} \left[A_1 e^{x\sqrt{yZ}} - A_2 e^{-x\sqrt{yZ}} \right] \longrightarrow ⑤$$

At receiving end $x=0$, $V=V_R$ and $I=I_R$

Putting these values in eqn ④ and ⑤ we get,

$$V_R = A_1 + A_2 \longrightarrow ⑥$$

$$I_R = \sqrt{\frac{y}{Z}} \left[A_1 - A_2 \right] \longrightarrow ⑦$$

Solving eqn ⑥ and ⑦ we get,

$$A_1 = \frac{V_R + I_R \sqrt{Z/y}}{2} = \frac{V_R + I_R Z_c}{2}$$

$$A_2 = \frac{V_R - I_R \sqrt{Z/y}}{2} = \frac{V_R - I_R Z_c}{2}$$

Where $Z_c = \sqrt{Z/y}$ = Characteristic impedance of the line

$\gamma = \sqrt{yZ}$ = Propagation constant.

Putting the values of A_1 and A_2 in eqn ④ and ⑤ we get

$$V = \left(\frac{V_R + I_R Z_c}{2} \right) e^{x\gamma} + \left(\frac{V_R - I_R Z_c}{2} \right) e^{-x\gamma} \longrightarrow ⑧$$

and $$I = \left(\frac{\frac{V_R}{Z_c} + I_R}{2} \right) \times e^{x\gamma} - \left(\frac{\frac{V_R}{Z_c} - I_R}{2} \right) \times e^{-x\gamma} \longrightarrow$$

Eqn ⑧ and ⑨ give the rms values of V and I.

The Long transmission Line: Interpretation of the Equations.

From equations ⑧ and ⑨,

$$V = \left(\frac{V_R + I_R Z_c}{2}\right) e^{x\gamma} + \left(\frac{V_R - I_R Z_c}{2}\right) e^{-x\gamma}$$

and
$$I = \left(\frac{\frac{V_R}{Z_c} + I_R}{2}\right) e^{x\gamma} - \left(\frac{\frac{V_R}{Z_c} - I_R}{2}\right) e^{-x\gamma}$$

Where $\gamma = \sqrt{yz}$ = Propagation Constant = $\alpha + j\beta$.

α = attenuation constant = It is a measure of spatial rate of decay of wave in the medium.

β = phase angle.

$Z_c = \sqrt{\frac{z}{y}}$ = Characteristic impedance of the line $= \sqrt{\frac{j\omega L}{G + j\omega C}}$

Put $\gamma = \alpha + j\beta$ in the above two equations, we get,

$$V = \left(\frac{V_R + I_R Z_c}{2}\right) e^{\alpha x} \cdot e^{j\beta x} + \left(\frac{V_R - I_R Z_c}{2}\right) e^{-\alpha x} \cdot e^{-j\beta x} \longrightarrow (10)$$

and
$$I = \left(\frac{\frac{V_R}{Z_c} + I_R}{2}\right) e^{\alpha x} \cdot e^{j\beta x} - \left(\frac{\frac{V_R}{Z_c} - I_R}{2}\right) e^{-\alpha x} \cdot e^{-j\beta x} \longrightarrow (11)$$

The 1st term in eqn (10), $\left(\frac{V_R + I_R Z_c}{2}\right) e^{\alpha x} \cdot e^{j\beta x}$ increases in magnitude and advances in phase ~~as~~ when we move from receiving end to sending end. Also magnitude of that term decreases and phase is retarded when we move from sending end to receiving end.

This is the characteristic of a traveling wav
The 1st term of eqn ⑩ is called incident voltage

The 2nd term in eqn ⑩, $\left(\frac{V_R - I_R Z_c}{2}\right) e^{-\alpha x} \cdot e^{-j\beta x}$ decreases in magnitude and retarded in phase when we move from receiving end to sending end. This 2nd term in eqn ⑩ is called reflected voltage

At any point along the line the voltage is the sum of incident and reflected voltages at that point

Similarly 1st term of eqn ⑪ represents incident current and 2nd term represents reflected current. At any point along the line the current is the sum of incident and reflected current.

→ If $V_R = I_R Z_c$ (i.e a line is terminated in its characteristic impedance Z_c) then the 2nd term of eqn ⑩ and ⑪ is zero and there is no reflected wave of either voltage or current.

"A line terminated in its characteristic impedance is called flat line or infinite line"

A line of infinite length can not have a reflected wave

→ Power lines are not terminated in their characteristic impedance Z_c.

For a single ckt overhead line Z_c is 400 Ω
For two circuits in parallel Z_c is 200 Ω
The phase angle of Z_c is usually betn 0 and $-15°$.

Bundle conductors have low inductance(L) and high capacitance (C), so they have lower value of Z_c. [Compared to single conductor per phase

→ In Power system, characteristic impedance 'Z_c' is called Surge impedance. The term surge impedance is usually reserved for lossless line.

We know that $Z_c = \sqrt{\frac{jwL}{G+jwC}}$

In lossless line $G = 0$.

$\therefore Z_c = \sqrt{L/C}$

→ Propagation constant $\gamma = \sqrt{yz} = \alpha + j\beta$.

In lossless line $\alpha = 0$.

So $\gamma = j\beta$

$\Rightarrow \gamma = \frac{jw\sqrt{LC}}{l}$

where l = length of the transmission line.

Note.

$\gamma = \sqrt{jw\mu(\sigma + jw\epsilon)}$

For loss less line $\sigma = 0$.

$\therefore \gamma = \sqrt{jw\mu(0 + jw\epsilon)} = jw\sqrt{\mu\epsilon}$

We know velocity $= \frac{1}{\sqrt{\mu\epsilon}}$

$\Rightarrow \frac{distance}{time} = \frac{1}{\sqrt{\mu\epsilon}}$

$\Rightarrow \frac{l}{\sqrt{LC}} = \frac{1}{\sqrt{\mu\epsilon}}$

$\Rightarrow \sqrt{\mu\epsilon} = \sqrt{LC}/l$

$\therefore \gamma = jw\sqrt{\mu\epsilon} = \frac{jw\sqrt{LC}}{l}$

→ With high frequencies or with surges due to lightning, losses are neglected and surge impedance becomes important.

→ Surge impedance loading (SIL) of a line is the Power delivered by a line to a Purely resistive load equal to Surge impedance.

When loaded, the line supplies a current I_L.

$$\therefore I_L = \frac{V_L}{\sqrt{3}\, Z_c} = \frac{V_L}{\sqrt{3}\sqrt{L/C}} \text{ Ampere.}$$

where V_L = Line to line voltage.

Power delivered by a line = Surge impedance loading (SI

$$= \sqrt{3}\, V_L I_L \cos\phi$$

$$= \sqrt{3}\, V_L \cdot \frac{V_L}{\sqrt{3}\sqrt{L/C}} \cdot 1$$

{ For p
resistan
load cos φ

$$= \frac{V_L^2}{\sqrt{L/C}} \text{ watt}$$

→ Let λ = wave length of wave

and f = frequency of wave.

$$\therefore \text{Phase constant } \beta = \frac{2\pi}{\lambda} \rightarrow ⑫$$

Let v = velocity of Propagation of a wave.

$$\beta = \frac{\omega}{v} = \frac{2\pi f}{v} = \frac{2\pi f}{\text{distance}/\text{time}} = \frac{2\pi f}{l/\sqrt{LC}}$$

$$\Rightarrow \beta = \frac{2\pi f\sqrt{LC}}{l} \rightarrow ⑬$$

Note

Dimension of $L = (M^1 L^2 T^{-2} A^{-2})$

Dimension of $C = (M^{-1} L^{-2} T^4 A^2)$

Dimension of $LC = T^2$

so $\sqrt{LC} = T$ = time.

OR

$$\sqrt{LC} = \sqrt{\frac{L}{R} \cdot RC} = \sqrt{\text{time} \times \text{time}}$$

= time.

From equation ⑫ and ⑬ we get

$$\frac{2\pi}{\lambda} = \frac{2\pi f\sqrt{LC}}{l}$$

$$\Rightarrow \lambda = \frac{l}{f\sqrt{LC}} \text{ meter}$$

The Long transmission line :

Hyperbolic form of the equations :

From equations ⑧ and ⑨,

$$V = \left(\frac{V_R + I_R Z_c}{2}\right) e^{x\gamma} + \left(\frac{V_R - I_R Z_c}{2}\right) e^{-x\gamma}$$

$$= V_R\left(\frac{e^{\gamma x} + e^{-\gamma x}}{2}\right) + I_R Z_c\left(\frac{e^{\gamma x} - e^{-\gamma x}}{2}\right)$$

$$= V_R \cosh \gamma x + I_R Z_c \sinh \gamma x \longrightarrow (14)$$

Note

$$\cosh\theta = \frac{e^{\theta} + e^{-\theta}}{2}$$

$$\sinh\theta = \frac{e^{\theta} - e^{-\theta}}{2}$$

Similarly, $I = \left(\frac{\frac{V_R}{Z_c} + I_R}{2}\right) e^{x\gamma} - \left(\frac{\frac{V_R}{Z_c} - I_R}{2}\right) e^{-x\gamma}$

$$= I_R\left(\frac{e^{\gamma x} + e^{-\gamma x}}{2}\right) + \frac{V_R}{Z_c}\left(\frac{e^{\gamma x} - e^{-\gamma x}}{2}\right)$$

$$= I_R \cosh \gamma x + \frac{V_R}{Z_c} \sinh \gamma x \longrightarrow (15)$$

where x is the distance from receiving end to the element of length dx.

Let $x = l$, to obtain the voltage & current at sending en

So Equations (14) and (15) becomes,

$$V_S = V_R \cosh \gamma l + I_R Z_c \sinh \gamma l \longrightarrow (16)$$

and $I_S = I_R \cosh \gamma l + \frac{V_R}{Z_c} \sinh \gamma l \longrightarrow (17)$

But We Know, $V_S = A V_R + B I_R \longrightarrow$ ⑱

and $I_S = C V_R + D I_R \longrightarrow$ ⑲

Comparing the equations 16, 17 with 18, 19, we

$A = \cosh \gamma l$, $B = Z_c \sinh \gamma l$

$C = \dfrac{\sinh \gamma l}{Z_c}$, $D = \cosh \gamma l$

→ If we move from sending end to receiving end then eqn ⑯ and ⑰ will be,

$V_R = V_S \cosh \gamma l - I_S Z_c \sinh \gamma l \longrightarrow$ ⑳

$I_R = I_S \cosh \gamma l - \dfrac{V_S}{Z_c} \sinh \gamma l. \longrightarrow$ ㉑

Note

Here put $V_S = V_R$ and $l = -l$ in eqn ⑯ and ⑰

* * *

For Problem: $\cosh \theta = \dfrac{e^{\theta} + e^{-\theta}}{2}$, $\sinh \theta = \dfrac{e^{\theta} - e^{-\theta}}{2}$

Here γ is a complex number. i.e $\gamma = \alpha + j\beta$

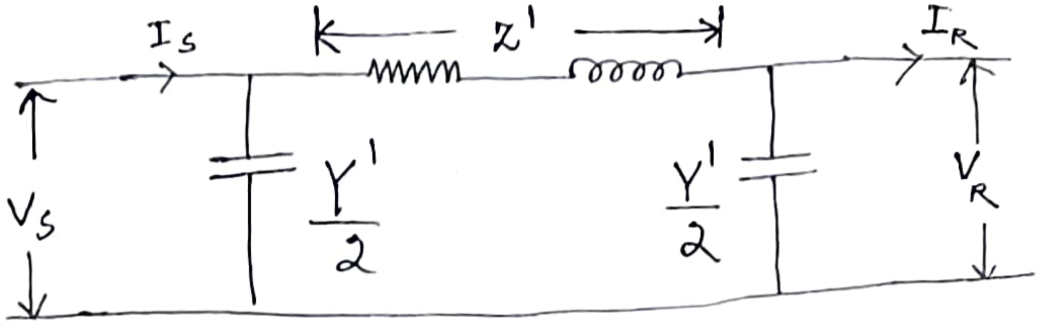
$$\therefore \cosh \gamma = \cosh(\alpha + j\beta) = \frac{e^{\alpha + j\beta} + e^{-\alpha - j\beta}}{2} = \frac{e^{\alpha} \cdot e^{j\beta} + e}{2}$$

$$= \frac{1}{2}\left[e^{\alpha} \angle \beta + e^{-\alpha} \angle -\beta\right]$$

$$\sinh \gamma = \sinh(\alpha + j\beta) = \frac{e^{\alpha} \cdot e^{j\beta} - e^{-\alpha} \cdot e^{-j\beta}}{2} = \frac{1}{2}\left[e^{\alpha} \angle \beta - e^{-\alpha}\right.$$

The Equivalent circuit of a Long line :-

→ To find the equivalent circuit of a long transmission line, Let us assume a π circuit similar to medium transmission line as shown in fig.



→ $$V_S = \left(\frac{Z'Y'}{2} + 1\right) V_R + Z' I_R \longrightarrow ㉒$$

But we know that $V_S = V_R \cosh \gamma l + I_R Z_C \sinh \gamma l \rightarrow ㉓$

Comparing Eqn ㉒ and ㉓ we get,

$$Z' = Z_c \sinh \gamma l$$

$$\left[\because Z_c = \sqrt{\frac{z}{y}}\right]$$

z = impedance per unit length

$$\Rightarrow Z' = \sqrt{\frac{z}{y}} \cdot \sinh \gamma l$$

$$\Rightarrow z' = \frac{z \cdot l}{\sqrt{zy} \cdot l} \sinh \gamma l$$

$$\Rightarrow Z' = \frac{Z}{\gamma l} \cdot \sinh \gamma l$$

Where $Z = zl$ = total series impedance of the line

and $\frac{\sinh \gamma l}{\gamma l}$ = a ~~[illegible]~~ factor by which the series impedance of nominal π must be multiplied to convert the nominal π to the equivalent π.

→ If γl is very small then $\sinh\gamma l \rightarrow \gamma l$
Above eqn becomes, $Z' = Z$

∴ The nominal π represents the medium-lengt transmission line quite accurately, in so as the series arm is concerned.

→ If we compare eqn ㉒ and ㉓,

$$\left(\frac{Z'Y'}{2}+1\right) = \cosh\gamma l.$$

$$\Rightarrow \left(\frac{Z_c\sinh\gamma l\cdot Y'}{2}+1\right) = \cosh\gamma l.$$

$\left[∵ Z_c \sinh\right.$

$$\Rightarrow \frac{Y'}{2} = \frac{1}{Z_c}\left(\frac{\cosh\gamma l-1}{\sinh\gamma l}\right)$$

$$\Rightarrow \frac{Y'}{2} = \frac{1}{Z_c}\cdot\tanh\frac{\gamma l}{2}$$

$\left\{∵ \tanh\frac{\gamma l}{2} = \frac{\cosh\gamma l-1}{\sinh\gamma l}\right.$

$$\Rightarrow \frac{Y'}{2} = \frac{1}{\sqrt{\frac{Z}{Y}}}\cdot\tanh\frac{\gamma l}{2}$$

{∵ Z = impedance per unit
Y = shunt admittance p
unit leng

$$\Rightarrow \frac{Y'}{2} = \frac{1}{\sqrt{\frac{ZY}{Y^2}}}\tan\frac{\gamma l}{2} = \frac{1}{\frac{\sqrt{zy}}{y}}\tan\frac{\gamma l}{2} = \frac{1}{\frac{\gamma}{y}}\tan\frac{\gamma l}{2}$$

$\left[∵ \sqrt{zy} = \gamma\right.$

$$\Rightarrow \frac{Y'}{2} = \frac{y}{\gamma}\tan\frac{\gamma l}{2} = \frac{yl}{\gamma l}\tan\frac{\gamma l}{2} = \frac{Y}{\gamma l}\tan\frac{\gamma l}{2}$$

$\left[∵ yl = Y\right]$

$$\Rightarrow \frac{Y'}{2} = \frac{Y/2}{\gamma l/2}\tan\frac{\gamma l}{2}$$

$$\Rightarrow \frac{Y'}{2} = \frac{Y}{2}\left(\frac{\tanh\frac{\gamma l}{2}}{\frac{\gamma l}{2}}\right)$$

{∵ $Y = yl$ = total sh admittance of th line.

(29)

$\frac{\tan \frac{\gamma l}{2}}{\gamma l/2}$ = a factor by which the shunt admittance of nominal π must be multiplied to convert the nominal π to the equivalent π.

→ If $\frac{\gamma l}{2}$ is very small then $\tan \frac{\gamma l}{2} \rightarrow \frac{\gamma l}{2}$.

So Above eqn. becomes $\frac{Y'}{2} = \frac{Y}{2}$.

Power flow through a transmission line :-

→ We know that, $V_s = A V_R + B I_R$

$$\Rightarrow I_R = \frac{V_s - AV_R}{B} = \frac{V_s}{B} - \frac{AV_R}{B}$$

Let $A = |A| \angle \alpha$, $B = |B| \angle \beta$

$V_R = |V_R| \angle 0°$, $V_s = |V_s| \angle \delta$

$$\therefore I_R = \frac{|V_s| \angle \delta}{|B| \angle \beta} - \frac{|A| \angle \alpha \times |V_R| \angle 0°}{|B| \angle \beta}$$

$$\Rightarrow I_R = \frac{|V_s|}{|B|} \angle \delta - \beta - \frac{|A||V_R|}{|B|} \angle \alpha - \beta$$

Complex power at the receiving end is $= V_R I_R^*$

$$I_R^* = \frac{|V_s|}{|B|} \angle \beta - \delta - \frac{|A||V_R|}{|B|} \angle \beta - \alpha$$

$$\therefore \text{Complex power} = V_R I_R^* = \frac{|V_s||V_R|}{|B|} \angle \beta - \delta - \frac{|A||V_R|^2}{|B|} \angle \beta - \alpha$$

Also $V_R I_R^* = P_R + jQ_R = \frac{|V_S||V_R|}{|B|}\angle \beta - \delta - \frac{|A||V_R|^2}{|B|}$

$\Rightarrow P_R + jQ_R$

Over head Line Insulators :

The insulators for overhead lines provide insulation to the power conductor from the ground. The insulators are connected to the cross arm of the supporting structure and the power conductor passes through the clamp of the insulator.

→ Properties of over head line insulator are,

(i) high mechanical strength so as to bear the load due to the weight of line conductors, wind force and ice loading.

(ii) high relative permittivity so as to provide high dielectric strength.

(iii) high insulation resistance in order to prevent leakage of currents to earth.

(iv) ability to withstand large temp variations.

→ The materials for insulators are toughened glass and porcelain.

(i) Porcelain :- ⇒ It is produced by firing at a controlled temp. a mixture of kaolin, feldspar and quartz.

➡ It is mechanically stronger than glass.

➡ It gives less trouble from leakage and less susceptible to temp. variations and its surface is not affected by dirt deposits.

→ It is difficult to manufacture homogeneous Porcelain and therefore, for a Particular operating voltage two, three or more pieces Construction is adopted in which each piece is glazed separately and then they are cemented together.

(ii) <u>Toughened glass :-</u>

→ It is cheaper than Porcelain.

→ It has high dielectric strength

→ It has low coefficient of thermal expansion.

→ It is a transparent substance, so it is very easy to detect any flaw like trapping of air etc.

→ The main disadvantage of glass is that moisture more readily condenses on its surface and facilitates the accumulat of dirt deposits, thus giving a high surface leakage.

Types of insulators :- There are three types of insulators used for overhead lines

(i) Pin type

(ii) Suspension type.

(iii) Strain type.

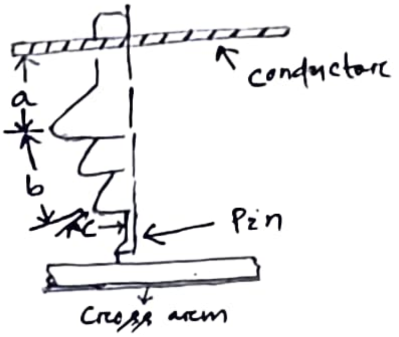
(i) Pin type insulators :-

→ Pin type insulators are used for transmission and distribution of electric power at voltages upto 33 KV.

→ It is small, simple in construction and cheap.

→ The conductor is bound into a groove on the top of the insulator which is cemented on to a galvanized steel pin attached to the crossarm on the pole or tower.

→ The electrical breakdown of the insulator can occur either by flash over or puncture.



In flashover, an arc occurs betn the line conductor and insulator pin and discharge jumps across the air gaps.
In fig a+b+c represents arcing distance for the insulator.
In case of flash over, the insulator will continue to act in its proper capacity unless extreme heat produced by the arc destroys the insulator.

In puncture, the discharge occurs from conductor to pin through the body of the insulator. When such breakdown involved then the insulator is peremanently destroyed due to excessive heat. To avoid puncture, sufficient thickness of porcelain is provided in the insulator.

The reatio of puncture strength to flashover voltage is called safty factor.

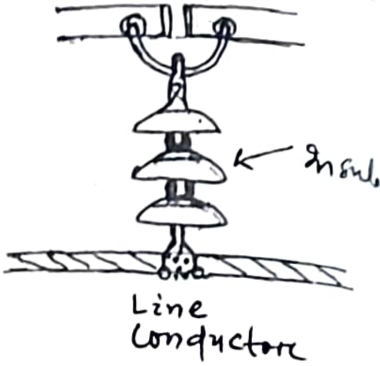
earth or tower is called shunt capacitance.

$$\therefore \quad \text{Safty Factor} = \frac{\text{Puncture Strength}}{\text{Flash over voltage.}}$$

For Pin insulator, safty factor is about 10.

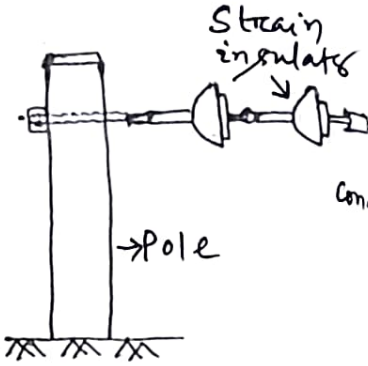
(ii) Suspension type insulators :-

→ It is used for high voltages i.e greater than 33 KV



→ Suspension type insulators consist of a number of Porcelain discs Connected in series by metal links in the form of a string. The conductor is suspended at the bottom end of this string while the other end of the string is secured to the cross arm of the tower. Each unit (or disc is designed for low voltage (say 11 KV). For Example, if the working voltage is 66 KV, then six disc are connected in series.

(iii) Strain insulators :-



→ When there is a dead end of the line or there is corner, the line is subjected to greater tension. Inorder to relieve the line of excessive tension, Strain insulators are used.

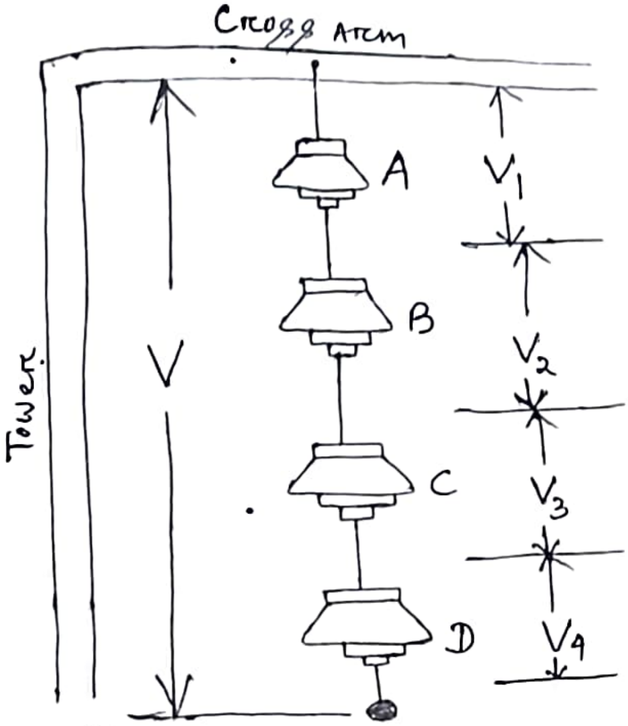
→ Strain insulators consist of an assembly of suspension insulators as shown in fig. The discs of strain insulators used in vertical plane.

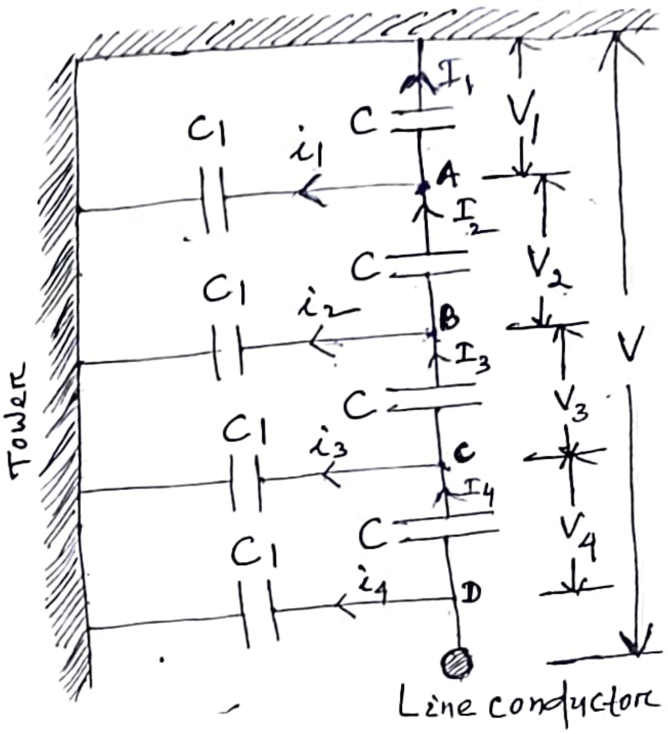
Voltage Distribution Over insulator string :-

→ Let us consider overhead lines operating at high voltages (i.e 33 KV and above) use many discs connected in series as shown in fig.

, The whole unit formed by connecting number of discs in series known as string of insulators.

The Line Conductor is connected to the bottom disc and top disc is connected to the cross arm of tower.





→ Voltage applied between line conductor and earth does not distribute uniformly across the discs.

→ Each string insulator unit behaves like a capacitor having a dielectric medium between two metallic Parts (Pin & cap)

The capacitance due to two metal fittings on either side of an insulator is known as mutual capacitance.

→ A capacitance between metal fitting of each unit and earth or tower is called shunt capacitance.

→ Due to shunt capacitance, charging current is not same through all discs of the strin so voltage across individual units will be different.

→ String efficiency = $\frac{\text{Voltage across the st}}{\text{n × voltage across the lower most unit}}$

where n = no. of units in the string.

Also String efficiency = $\frac{\text{Flash over voltage of the string}}{\text{n × flash over voltage of one unit.}}$

Let, C = mutual capacitance.
C_1 = Shunt capacitance.
V_1 = voltage across 1st unit (nearest to cross arm
V_2 = " " 2nd unit.
V_3 = " " 3rd unit
V_4 = " " 4th unit (nearest to line cond

V = voltage between conductor and earth.

Let $\frac{C_1}{C} = K \Rightarrow C_1 = KC.$

$$\left[I = C \cdot \frac{dv}{dt} \quad I = C \cdot \frac{V}{t} \right]$$

Applying KCL to node-A,

$$I_2 = I_1 + i_1$$

$$\Rightarrow \omega C V_2 = \omega C V_1 + \omega C_1 V_1$$

㊲

$$\Rightarrow \omega C V_2 = \omega C V_1 + \omega K C V_1 \quad \{\because C_1 = KC\}$$

$$\Rightarrow V_2 = V_1(1+K) \longrightarrow (1)$$

Applying KCL to node B,

$$I_3 = I_2 + i_2$$

$$\Rightarrow \omega C V_3 = \omega C V_2 + \omega C_1 (V_1 + V_2)$$

{∵ Voltage across the second shunt capacitance C_1 from top $= V_1 + V_2$.

$$\Rightarrow \omega C V_3 = \omega C V_2 + \omega K C (V_1 + V_2)$$

$$\Rightarrow V_3 = V_2 + K(V_1 + V_2).$$

$$= K V_1 + V_2 (1+K)$$

$$\Rightarrow V_3 = K V_1 + V_1 (1+K)(1+K) \quad \{\because V_2 = V_1 (1+K)\}$$

$$\Rightarrow V_3 = V_1 (1 + 3K + K^2) \longrightarrow (2)$$

Applying KCL to node c,

$$I_4 = I_3 + i_3$$

$$\Rightarrow \omega C V_4 = \omega C V_3 + \omega C_1 (V_1 + V_2 + V_3)$$

$$\Rightarrow \omega C V_4 = \omega C V_1 (1 + 3K + K^2) + \omega K C \left[V_1 + V_1 (1+K) + V_1 (1 + 3K + K^2)\right]$$

$$\Rightarrow V_4 = V_1 (1 + 6K + 5K^2 + K^3) \longrightarrow (3)$$

Final voltage between line conductor & earth,

$$V = V_1 + V_2 + V_3 + V_4$$

$$= V_1 + V_1 (1+K) + V_1 (1 + 3K + K^2) + V_1 (1 + 6K + 5K^2 + K^3)$$

$$V = V_1 (4 + 10K + 6K^2 + K^3) \longrightarrow (4)$$

From eqn ① $V_1 = \frac{V_2}{1+k}$ → (5)

From eqn (2) $V_1 = \frac{V_3}{1+3K+k^2}$ → (6)

From eqn ③ $V_1 = \frac{V_4}{1+6k+5k^2+k^3}$ → ⑦

From eqn ④, $V_1 = \frac{V}{4+10k+6k^2+k}$ → ⑧

∴ From eqn ⑤, ⑥, ⑦ & ⑧ $V_1 = \frac{V_2}{1+K} = \frac{V_4}{1+6k+5k^2+k^3} = \frac{V}{4+10k+ \; +k}$

$= \frac{V_3}{1+3k+k^2}$

$$\therefore \quad V_4 = \left(\frac{1+6k+5k^2+k^3}{4+10k+6k^2+k^3}\right) V \text{ volt.}$$

$$\% \text{ string efficiency} = \frac{V}{n V_4} \times 100$$

$$= \frac{V}{4 \times \left(\frac{1+6k+5k^2+k^3}{4+10k+6k^2+k^3}\right) V} \times 100$$

(∵ n = 4, greatest voltage across any unit i

$$\therefore = \frac{4+10k+6k^2+k^3}{4(1+6k+5k^2+k^3)} \times 100.$$

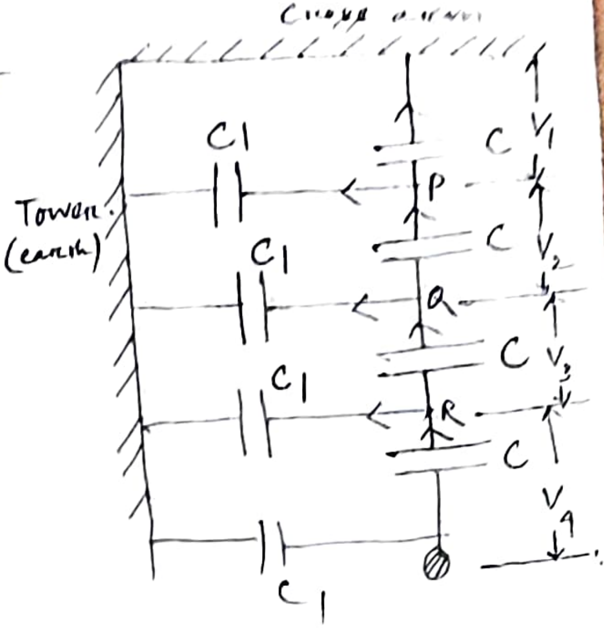
Excercise
Problem 4.2
B.R.GUPTA.

A String insulator of 66 kV line has 4 discs. The capacitance fro each joint to tower is 25% of the self capacitance of each unit. Find the voltage distribution across the units and string efficiency.

(39)

Solution :-

Given $C_1 = 0.25 C$, $K = 0.25$



Apply KCL to node-P :

$$\omega C V_2 = \omega C V_1 + \omega C_1 V_1$$

$$\Rightarrow C V_2 = C V_1 + 0.25 C V_1$$

$$V_2 = V_1 + 0.25 V_1$$

$$\Rightarrow V_2 = 1.25 V_1$$

$\Rightarrow$

Apply KCL to node-Q

$$\omega C V_3 = \omega C V_2 + \omega C_1 (V_1 + V_2)$$

$$\Rightarrow C V_3 = C V_2 + 0.25 C (V_1 + V_2)$$

$$\Rightarrow V_3 = V_2 + 0.25 V_1 + 0.25 V_2$$

$$\Rightarrow V_3 = 1.25 V_2 + 0.25 V_1$$

$$\Rightarrow V_3 = 1.25 (1.25 V_1) + 0.25 V_1$$

$$\Rightarrow V_3 = 1.8125 V_1$$

Applying KCL to node-R

$$\omega C V_4 = \omega C V_3 + \omega C_1 (V_1 + V_2 + V_3)$$

$$\Rightarrow C V_4 = C V_3 + 0.25 C (V_1 + V_2 + V_3)$$

$$\Rightarrow V_4 = 1.8125 V_1 + 0.25 (V_1 + 1.25 V_1 + 1.8125 V_1)$$

$$\Rightarrow V_4 = 2.8281 V_1$$

But $V_1 + V_2 + V_3 + V_4 = \frac{66 \times 10^3}{\sqrt{3}} = 38.1 \times 10^3$ volts.

$$\Rightarrow V_1 + 1.25 V_1 + 1.8125 V_1 + 2.8281 V_1 = 38.1 \times 10^3$$

$$\Rightarrow 6.9031 V_1 = 38.1 \times 10^3$$

$$\Rightarrow V_1 = 5.519 \times 10^3 = 5519 \text{ volts.}$$

$V_1 = 5519$ volts.

$V_2 = 1.25\,V_1 = 1.25 \times 5519 = 6898.75$ volts

$V_3 = 1.8125\,V_1 = 1.8125 \times 5519 = 10003.1875$ volts.

$V_4 = 2.8281\,V_1 = 2.8281 \times 5519 = 15608.28$ volts.

$$\text{String efficiency} = \frac{38.1\times10^{3} \times 100}{4\times15608.28} = 61.02\%$$

Methods to improve string efficiency

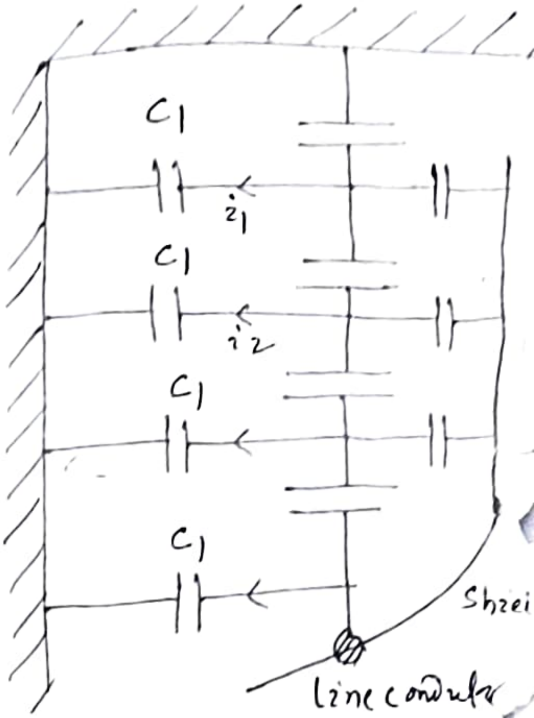
(i) Reduction in the shunt capacitance relative to the capacitance of each unit. This can done by increasing the length of cross ar As a result $K = \frac{C_1}{C}$ be reduced. But the limitat of cost and mechanical strength of line suppo do not allow the cross arms to be too long and it has been found that it is not Possibl to obtain the value of K less than 0.1.

(ii) Non-uniform distribution of voltage across an insulator string is due to leakage curre from the insulator pin to supporting struct This current can not be eliminated. However it Possible that discs of different capacities are used such that the Product of thei capacitive reactance and the current flowing through the respective unit is same.

(iii) By static shielding :-

In this method a guard or grading which usually takes the form of a large met ring surrounding the bottom unit and elect connected to the metal work at the bottom of this u and therefore to the line conductor.

The guard ring screens the bottom units, reduce their earth capacitance C_1 and introduces a number of capacitances between the line conductor and the various insulator unit caps. These capacitances are greater for bottom units and thus the voltages across them are reduced. With this method it is impossible to obtain an equal distribution of voltage but considerable improvements are possible.

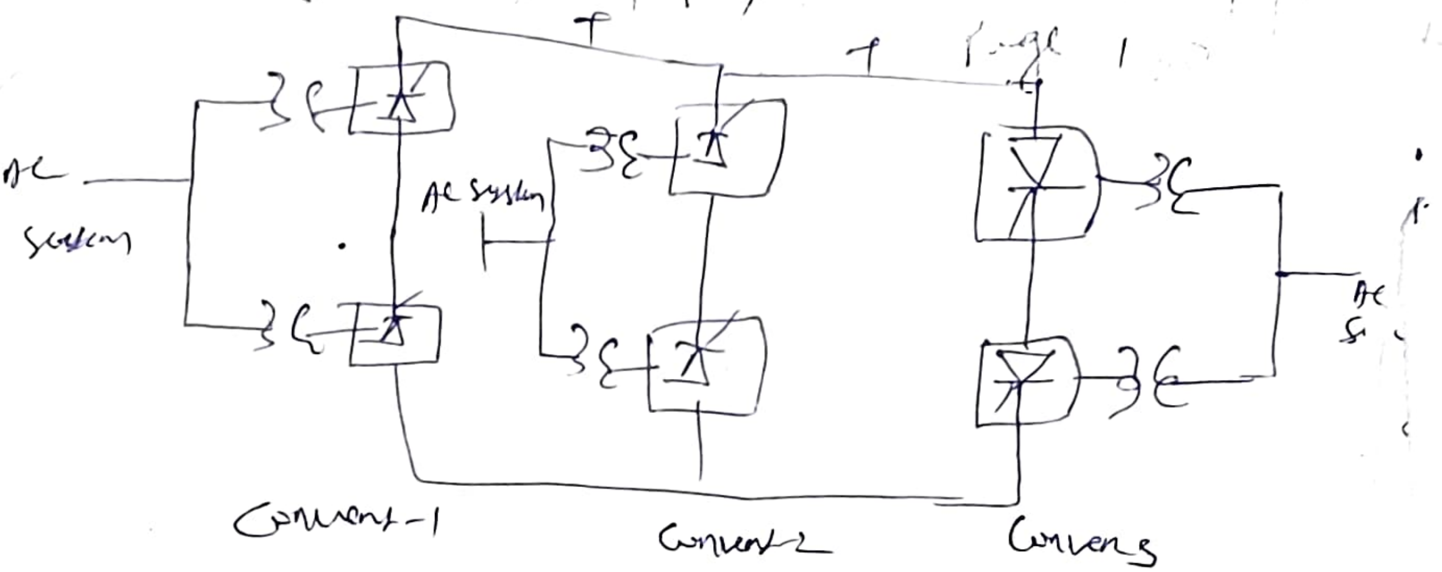


Insulator Failure:-

(Read by yourself [illegible])

Testing of Insulators:-

Read by yourself [illegible] page [illegible]



(3 terminal HVDC drawing)

Insulator failure) The failure of an insulator can be due to mechanical stresses, cracking, porosity, puncture & flashover.

Incorporating HVDC into AC systems

An HVDC link can be incorporated in an ac system in a number of ways. Each of these schemes is special in its own way & serves a specific purpose which is different from that of the other schemes.

1. Two terminal DC link for point to point transmission: A two terminal dc link using overhead lines is used to transmit large blocks of relatively clear power from remote sources (generally hydro electric) to load centres situated far away from the generating station. Such a scheme enables the development of remote power generating sites.

2. Back to Back DC links: A back to back link consists of sending end & receiving end converter stations situated close to each other & the dc line is of zero or negligible length. Such schemes are used to couple ac systems of different frequency (i.e. 50 Hz & 60 Hz systems) or of different system controls (i.e. asynchronous systems).

3. DC link in parallel with AC link: Sometimes it is necessary to put a dc link with an existing ac line. Such a scheme is used to reinforce the ac link & it improve its stability.

4. Multi terminal DC links: All large ac power systems are essentially of multi-terminal variety. A multiterminal dc now is theoretically possible having a no. of rectifier & inverter stations. One of the essential requirements in a multiterminal dc scheme is use of the large dc cut breakers. The two terminals dc link doesn't need to be equipped with dc cut breakers because faults on the line can be cleared by the converter control action. However a multi-terminal scheme needs dc cut breakers.

HVDC Systems in INDIA:- HVDC has been incorporated in the Indian grid also. About 5800 ckt km of HVDC lines are in operation.

1. Rihand – Delhi HVDC system (A ± 500kV, 1500MW, 810km bipolar
2. Vindhyachal HVDC back to back system
- Talcher - Kolar HVDC system (A 2000MW, ± 500kV, bipolar

EPTD 3

(1)

Module-3 Mechanical Design of 'Overhead Lines :-

Transmission and Distribution of electric Power from the generating Stations to Various Consumers is carried out with minimum Possible loss and disturbance. It can be achieved if the transmission and distribution system is so designed and constructed that it is an efficient, technically sound and reliable system.

→ The Line should have sufficient current carrying capacity so as to transmit the required Power over a given distance without an excessive voltage drop and overheating. The Line losses should be small and insulation of the line should be adequate to cope with the system voltage.

Line supports :-

→ The Supports for an overhead line must be capable of carrying the load due to the conductors and insulators together with the wind load on the support itself.

→ The supports generally used are wooden Poles, RCC poles, steel tubular poles and steel towers.

→ The main requirements of the line supports are

(i) high mechanical strength to withstand the weight of conductors and wind load.

(ii) Light in weight without the loss of mechanical strength.

(iii) Cheaper in cost.

(iv) Low maintenance cost

(v) Longer Life

(vi) Good looking

(vii) Easy accessibility for Painting a erection of line

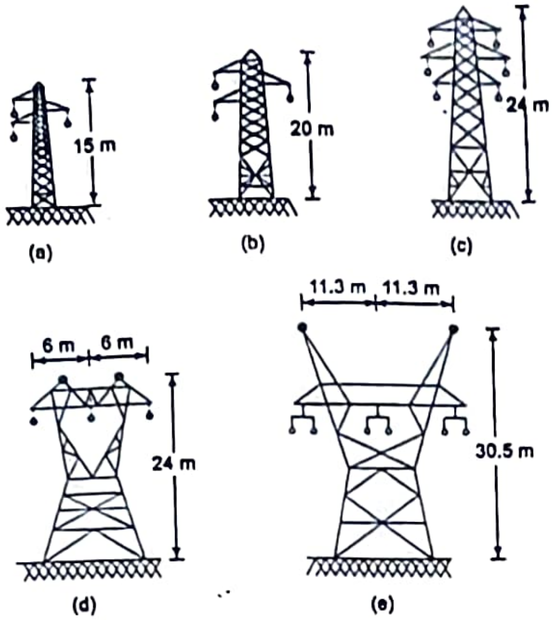
Wooden Poles :- Wooden poles are used for distri lines especially in areas wher ample supplies of good quality are available. Wood Poles ar Protected by an aluminium or zi cap at the top and bitumen coat over the Portion of the Pole in ground. For LV lines only one p is used but for 33 KV lines two p are used in A or H formation.

RCC Poles :- Poles made of reinforced cement con are stronger but more costly than wooden poles. They have very long life and little maintenance. They are used for distribution line upto 33 KV in urban areas. They are bulky a heavy.

Steel Tubular Poles :- They are most costly than RCC and wood poles. They ar light weight, high strength an long life. They are used for line upto 33 KV.

Steel towers :- Lines of 66 KV and above are supported on steel towers. They are fabricated from painted or galvanized angle sections which can be transported separately and the erection done on site. They have long life and high degree of reliability.

They can withstand very severe weather conditions. They are suitable for double circuits. Towers of many shapes and sizes are used for different types of lines as shown below.



Steel Towers (a) 33 kV, narrow base, single circuit, (b) 66 kV, b
single circuit (c) 132 kV, double circuit (d) 220 kV, Cat's head, s
(e) 400 kV, single circuit with two sub-conductors per phase

The height of the tower depends on the line voltage and span length. The legs of the towers are set in concrete foundations.

Types of Steel towers :— ~~they~~

Steel towers can be broadly classified as
(i) tangent towers
(ii) deviation towers.

For 33 KV spacing in 190 mm and so on.

(i) Tangent towers :-

→ In these towers the stress is due t weight of the line, ice and wind

→ The base of the tower may be sq rectangular. suspension type insulators are used with these towers.

(ii) Deviation towers :-

→ They are used at points where th transmission line changes direction.

→ They have broader base, stronger me and are costlier as compared to to towers. Strain insulators are used these towers.

Cross Arms :-

→ The function of cross-arms is to k the conductors at a safe distance each other and from the pole.

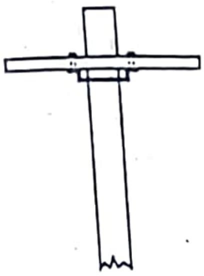
→ Cross-arm is a cross-piece fitt the pole top end portion by mean of brackets (known as pole brackets) supporting insulators.

→ Wooden cross-arms are commonly emp on 11 KV and 33 KV lines.

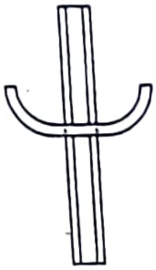
reliability.

Wooden cross-arms are preferred owing to their insulating property which provides safety to line staff and minimizes flash-over due to birdage.

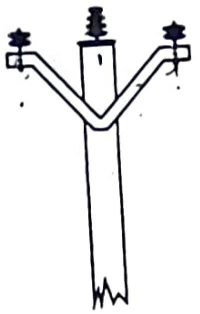
Various cross arms are shown below.



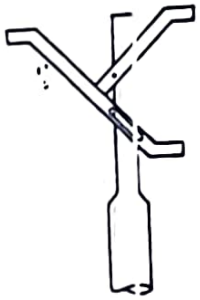
(a) MS Channel or Wooden Cross-arm



(b) U-Shaped Cross-arm



(c) V-Shaped Cross-arm



(d) Zig-zag C oss-arm

→ Steel cross-arms are stronger and are generally used on steel poles.

→ The cross arm is fixed to the pole in such a way that the load of the conductors is taken by the cross-arm and not the clamp or bolt that fixes the cross-arm to the pole.

→ In order to prevent arcing, the construction of the cross-arms should be such that under the worst conditions, the spacing between conductors, when swinging would never be less than that given below:

For 66 KV spacing is 76 mm
For 11 KV spacing is 101 mm
For 33 KV spacing is 190 mm and so on.

Span, Conductor Configuration, Sp and clearances :-

→ A longer span means a smaller number of but the towers are taller and more cos The higher the operating voltage, greater shoul span to reduce the high cost of insulators

→ The insulators constitute the weakest part a transmission line and reduction of numb of towers increases the reliability of the

→ For every proposed line there is a defini length of span which will give the minimu cost of the line.

→ Modern high voltage lines have spans betwe 200 and 400 m. For river and ravine crossin exceptionally long spans upto 800 m or 20 been satisfactory employed.

→ Many conductor configurations are used in prac In many cases flat horizontal and vertical configurations are used. A flat horizonta configuration means a lesser tower height but a wider right of way. A veritical configura means a taller tower and increased lighting hazards.

→ There must be some spacing between the conductors so that they do not come within sparking distance of each other even while swinging due to wind.

reliability.

3

(4)

→ There is an emprical formula used for spacing of aluminium conductor lines.

$$\text{Spacing} = \sqrt{S} + \frac{V}{150} \text{ metres.}$$

where S = sag in metres.
V = Line voltage in KV.

→ The Indian Electricity Rules specify the minimum clearance between the ground & conductor.

Clearance to ground	0.4 KV	11 KV	33 KV	66 KV	132 KV	400 KV
(i) across street in metre	5.8	5.8	6.1	6.1	6.1	8.4
ii) along street in metre	5.5	5.5	5.8	6.1	6.1	8.4
iii) Other areas in metre	4.6	4.6	5.2	5.5	6.1	8.4.

ue due to fall in temp.

Sag

Q:— The difference in level between the Points of supports and the lowest Point on the conductor is known as Sag.

f the Sag is Very Small Compared to the span then its Shape approximates a Parabola.

actors affecting the Sag:

Weight of Conductor;

- Heavier the Conductor, greater will be the sag.

Length of the Span;

Sag is directly Proportional to square of span length.

Working Tensile Strength;

Sag is inversely Proportional to the working tensile strength of conductor if temp., length of span are constant.

$$\text{working tensile strength of a conductor} = \frac{\text{Stress} \times \text{area of cross-section}}{\text{factor of safety}}$$

) **Temperature;**

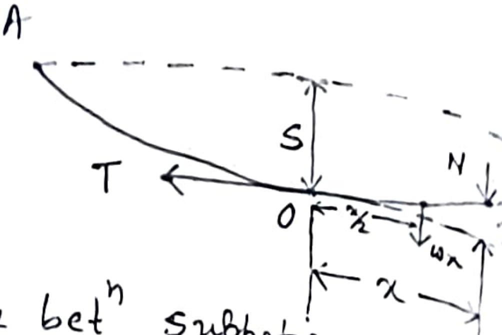
When temp. increases then length of conductor increases. so sag increases.

te (a) If sag is too high, more conductor material is required, more weight on the supports is to be supported, higher supports are necessary and there is a chance of greater swing amplitude due to wind.

(b) If sag is too low, there is more tension in the conductor and thus the conductor may break if any additional stress is to be taken such as due to vibration of line or due to fall in temp.

Sag calculation :-

Let us considere a conductore suspended between two equal level supports A and B as shown in fig.



S = sag = level difference betn suppots A and point 'o' of the conductor.

The exact shape of the conductore is a cate
Howevere, if the sag is small compared to
then its shape is a parabola.

Let, L = length of span i.e horizontal distance
Support

W = weight of the conductore per metre length

-- = tension in the conductor.

x = length of the conductor from mid point o

No. of forces acting on portion 'ON' are

(i) tension T acting at point 'o'

(ii) weight of conductor ON of length 'x' i.e Wx.

The weight Wx acting vertically downward at the C.G of conductor ON i.e at a distance $\frac{x}{2}$ from point 'o'.

Equating the moments about point N, we get

$$Ty = Wx\left(\frac{x}{2}\right)$$

$$\Rightarrow y = \frac{Wx^2}{2T}$$

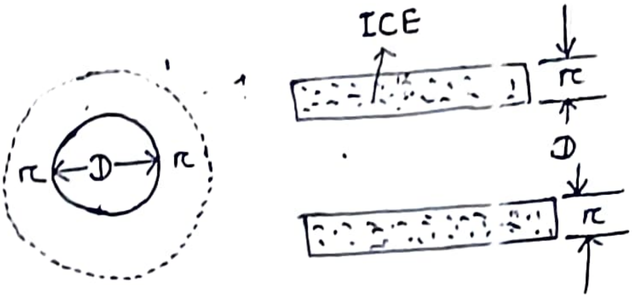
At point B, $y = s$ and $x = \frac{L}{2}$.

So $\text{Sag}(s) = \frac{W\left(\frac{L}{2}\right)^2}{2T}$

$$\Rightarrow \boxed{S = \frac{WL^2}{8T}} \longrightarrow ①$$

*

ffect of ice :—



he formation of ce coating on a ne conductor increases the weight and effective diameter of the conductor.

Total weight of the conductor acting vertically downward $= W_i + W_c$.

where W_i = weight of ice coating per metre length
W_c = " " conductor, in kg per metre length.

et D = diameter of the conductor.
r = radial thickness of ice coating

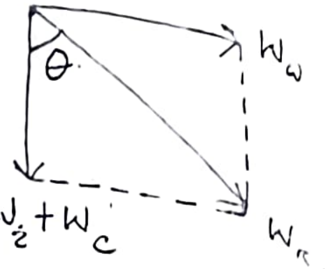
rom fig, Volume of ice coating per metre length of the conductor $= \frac{\pi}{4}\left[(D+2r)^2 - D^2\right]$
$= \pi r (D+r)\ m^3$.

Density of ice $= 920\ \text{kg}/m^3$.

∴ Weight of ice coating per metre length = (volume)(density)
$= \pi r (D+r)(920) = 2890.3\left[r(D+r)\right]\ \text{kg/m}$

Knowing W_i and W_c we can calculate sag by applying eqn ①.

Combined Effect of Wind and Ice :-



Let W_w = wind force in kg per metre length

$$= \left(\begin{array}{c}\text{Wind pressure}\\ \text{per } m^2 \text{ of}\\ \text{projected area}\end{array}\right) \times \left(\begin{array}{c}\text{Projected}\\ \text{area}\\ \text{per metre}\\ \text{length}\end{array}\right)$$

$$= (P) \times (D + 2r)$$

W_i = weight of ice coating in kg per metre le of the conductor.

W_c = weight of conductor in kg per metre length.

∴ Resultant weight per metre length of the Condu including ice-coating and wind force is

$$W_r = \sqrt{(W_i + W_c)^2 + W_w^2}$$

Knowing W_r, sag can be calculated from eqn ①.

$$\therefore \quad \text{Sag } (S) = \frac{W_r L^2}{8T} \longrightarrow ②$$

From fig $\theta = \cos^{-1}\left(\frac{W_i + W_c}{W_r}\right)$

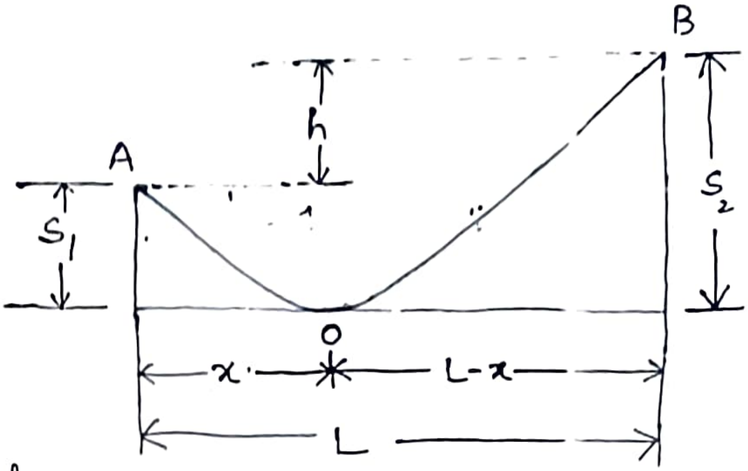
The angle 'θ' clearly indicates that the lowest po of the conductor is not vertically down bu away from it.

Equation ② given the slant sag expression.

$$\therefore \quad \text{Vertical Sag} = \frac{W_r L^2}{8T} \cos\theta . \longrightarrow ③$$

Calculate Sag if the Supports at Different levels:-

When the two supports A and B will be at different levels then the lowest Point 'O' of the conductor will not lie in the middle of the span.



Let x = Distance between lowest Point 'O' and support at low level.

$L - x$ = Distance between lowest Point 'O' and support at high level.

$\therefore S_1 = \frac{Wx^2}{2T}$ and $S_2 = \frac{W(L-x)^2}{2T}$

Difference in two level of supports is $S_2 - S_1 = h$

$$\Rightarrow \frac{W(L-x)^2}{2T} - \frac{Wx^2}{2T} = h$$

$$\Rightarrow WL^2 - 2WLx = 2Th$$

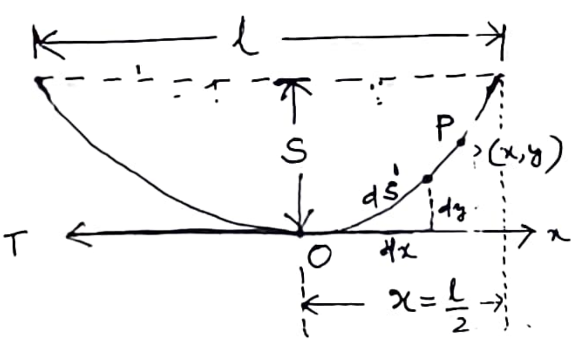
$$\Rightarrow \boxed{x = \frac{L}{2} - \frac{Th}{WL}}$$

Knowing 'x', S_1 and S_2 can be calculated.

Calculate total length of the conductor:-

Let s' = Curved distance from 'O' to P.

Consider a right angled triangle on the Parabola $y = ax^2$.



$$ds'^2 = dx^2 + dy^2$$

$$\Rightarrow \left(\frac{ds'}{dx}\right)^2 = 1 + \left(\frac{dy}{dx}\right)^2$$

$\Rightarrow \left(\frac{ds'}{dx}\right)^2 = 1+4a^2x^2$

$\Rightarrow ds' = (1+4a^2x^2)^{\frac{1}{2}} \cdot dx$

$\Rightarrow ds' = (1+2a^2x^2) \cdot dx$

$\left\{ \because y = ax^2,\quad \frac{dy}{dx} = 2ax,\quad \left(\frac{dy}{dx}\right)^2 = 4a^2x^2 \right\}$

Integrating both sides we get,

$$s' = x + \frac{2a^2x^3}{3} + K$$

$(1+x)^{1/2} = 1+\frac{x}{2}+$

When $x=0$, $s'=0$.

$\therefore K = 0$

Putting these values in the above expression we g

$$s' = x + \frac{2a^2x^3}{3} \longrightarrow (1)$$

Let Z = length of the line

If we put $x = l/2$ and $a = \frac{4S}{l^2}$ and curve distan

then $\frac{Z}{2} = \frac{L}{2} + 2\left(\frac{4S}{l^2}\right)^2 \times \frac{l^3}{8\times 3}$

$\left\{ \because \right.$ Note:- $y = ax^2$

$\Rightarrow a = \frac{y}{x^2}$

Put, $y = S$, $x =$

$a = \frac{S}{l^2/4} =$

$$\Rightarrow \frac{Z}{2} = \frac{L}{2} + \frac{4}{3}\frac{S^2}{L}$$

$$\Rightarrow Z = L + \frac{8S^2}{3L}$$

$$\Rightarrow Z = L + \frac{8}{3l} \times \frac{W_r^2 L^4}{64T^2}$$

$\left\{ \because \right.$ Sag, $S = \frac{W_r l^2}{8T}$

Considering effect of wind and ice.

$$\Rightarrow \boxed{Z = l\left[1 + \frac{W_r^2 l^2}{24T^2}\right]}$$

quivalent Span :-

A section of an overhead line may consists of a number of spans of different lengths because the location of the towers depends upon the profile of land along which the transmission line is to be laid.

t, n = no of spans of length L_1, L_2, L_3 ---etc which are to be given an equivalent span L_e.

$$nL_e = \Sigma L$$

But We know that, $l = L\left[1+\frac{W_r^2L^2}{24T^2}\right] = \left[L+\frac{W_r^2L^3}{24T^2}\right]$

So .. $$n\left[L_e+\frac{W_r^2L_e^3}{24T^2}\right] = \sum L+\frac{W_r^2L^3}{24T^2}$$

$$\Rightarrow n\left[L_e+\frac{W_r^2L_e^3}{24T^2}\right] = \Sigma L + \Sigma\frac{W_r^2L^3}{24T^2}$$

$$\Rightarrow nL_e + n.\frac{W_r^2L_e^3}{24T^2} = \Sigma L + \frac{W_r^2}{24T^2}\Sigma L^3,$$

$$\Rightarrow nL_e + \frac{n.W_r^2L_e^3}{24T^2} = nL_e + \frac{W_r^2}{24T^2}\Sigma L^3$$

$$\Rightarrow nL_e^3 = \Sigma L^3$$

$$\Rightarrow L_e^2 = \frac{\Sigma L^3}{nL_e}$$

$$\Rightarrow L_e^2 = \frac{\Sigma L^3}{nL_e} = \frac{\Sigma L^3}{\Sigma L}$$

$$\Rightarrow L_c = \sqrt{\frac{L_1^3 + L_2^3 + L_3^3 + \cdots}{L_1 + L_2 + L_3}}$$

Knowing L_e the tension T to which al
Sections would be erected can be calcul

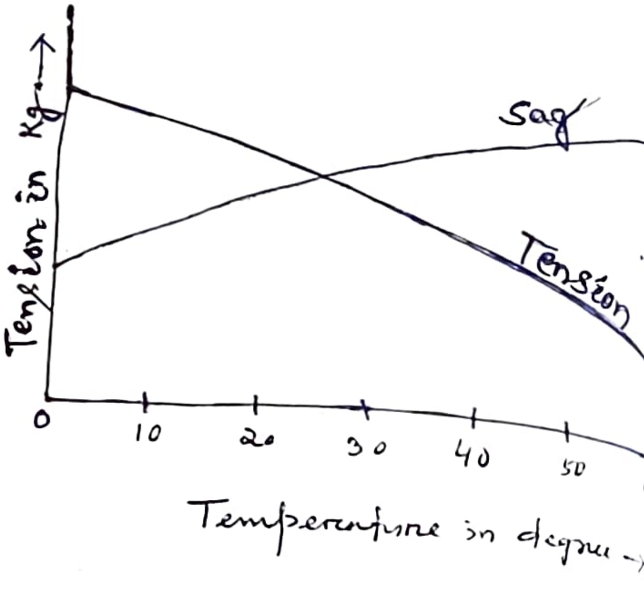
✓

Stringing chart :-

→ Stringing chart is helpful in knowing sag and tension at any temperature. T chart gives the data for sag to be a and the tension to be allowed at a partic temperature.

→ For preparation of stringing chart first of all calculate the sag and tension on conductor under the worst conditions i maximum wind pressure and minimum tem assuming a suitable factor of safty i fixing the maximum working tension on conductor.

→ Figure shows the plot of tension Vs temperature and sag vs temperature. this is called the Stringing chart.

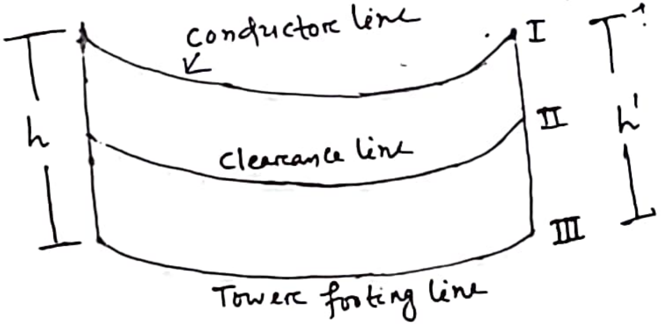


ag Temple :—

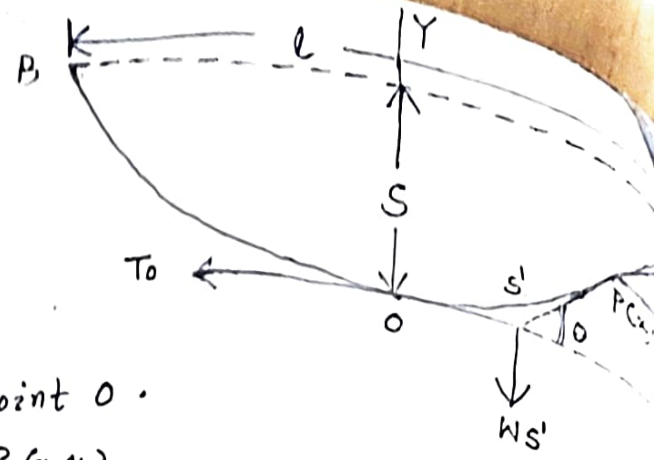
Sag temple is a very convenient method for locating the positions and height of towers/supports in the field.

The sag template is made on celluloid or tracing cloth. In sag template shown in fig the upper curve (I) represents the conductor line. The middle curve (II) is below the curve(I) by a uniform vertical distance equal to the desired minimum vertical clearance to ground. This clearance to ground is governed by the operating voltage and is given according to IE rules.

The lower curve (III) is below the upper curve by a uniform vertical distance equal to the height of a standard tower measured to the point of support of the conductor. If the location of the left tower has been decided then the location of right hand tower can be determined by adjusting the sag template, so that the conductor line passes through the point of support on the left hand tower and clearance line is tangent to ground at one or more points.



Catenary:



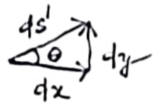
Consider a line supports at A and B at same level.

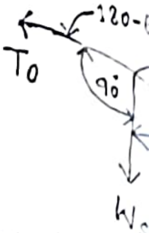
T_0 = tension at lowest point O.

T_p = tension at point $P(x,y)$.

S' = Length of portion OP

ws' = weight of portion OP





The portion 'op' is in equilibrium under the action of three forces T_0, T_p and Ws'.

So according to Lami's theorem, $\frac{T_p}{\sin 90} = \frac{Ws'}{\sin(180-\theta)}$

$$\Rightarrow \frac{T_p}{1} = \frac{Ws'}{\sin\theta} = \frac{T_0}{\cos\theta}$$

$$\Rightarrow T_p \sin\theta = Ws' \quad \text{and} \quad T_p \cos\theta = T_0$$

$$\therefore (T_p \sin\theta)^2 + (T_p \cos\theta)^2 = (Ws')^2 + (T_0)^2$$

$$\Rightarrow T_p^2 = T_0^2 + W^2 s'^2 \longrightarrow ①$$

Consider a small right angle triangle on the cate

$$(ds')^2 = dx^2 + dy^2$$

$$\Rightarrow \frac{ds'}{dx} = \sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{1+\tan^2\theta}$$

$\because$ tan θ =

But $$\frac{T_p \sin\theta}{T_p \cos\theta} = \frac{Ws'}{T_0}$$

$$\Rightarrow \tan\theta = \frac{Ws'}{T_0} \longrightarrow ②$$

$$\frac{ds'}{dx} = \sqrt{1+\frac{W^2 s'^2}{T_0^2}}$$

$$dx = \frac{ds'}{\left(1 + \frac{W^2 s'^2}{T_0^2}\right)^{0.5}}$$ ⑱

Integrating both sides we get,

$$x = \left(\frac{T_0}{W}\right) \sinh^{-1} \frac{Ws'}{T_0} + A \longrightarrow (3)$$

Where A is the Constant of integration.
At lowest Point of the Curve $x = 0$ and $s' = 0$

$\therefore$ $A = 0$.

Putting $A = 0$ in eqn ② we get,

$$x = \frac{T_0}{W} \sinh^{-1} \frac{Ws'}{T_0}$$

$$\Rightarrow \sinh^{-1} \frac{Ws'}{T_0} = \frac{Wx}{T_0}$$

$$\Rightarrow \frac{Ws'}{T_0} = \sinh \frac{Wx}{T_0}$$

$$\Rightarrow s' = \frac{T_0}{W} \sinh \frac{Wx}{T_0} \longrightarrow ④$$

Putting this value of s' in eqn ② we get,

$$\tan\theta = \frac{Ws'}{T_0}$$

$$\Rightarrow \tan\theta = \frac{W}{T_0} \cdot \frac{T_0}{W} \sinh \frac{Wx}{T_0}$$

$$\Rightarrow \tan\theta = \sinh \frac{Wx}{T_0}$$

$$\Rightarrow \frac{dy}{dx} = \sinh \frac{Wx}{T_0} \qquad \left\{ \because \tan\theta = \frac{dy}{dx} \right\}$$

$$\Rightarrow dy = \sinh \frac{Wx}{T_0} \cdot dx$$

Integrating both sides we get,

$$y = \frac{T_0}{W}\cosh\frac{Wx}{T_0} + B \longrightarrow ⑤$$

where B is the constant of integratio

At the lowest point $y=0$ and $x=0$.

$$\therefore \quad 0 = \frac{T_0}{W}\cosh\frac{W(0)}{T_0} + B$$

$$\Rightarrow \quad B = -\frac{T_0}{W}$$

Putting this value of B in eqn ⑤ we get,

$$y = \frac{T_0}{W}\cosh\frac{Wx}{T_0} - \frac{T_0}{W}$$

$$\Rightarrow y = \frac{T_0}{W}\left(\cosh\frac{Wx}{T_0} - 1\right) \longrightarrow ⑥$$

Putting the value of s' in eqn ① we get,

$$\therefore T_p^2 = T_0^2 + W^2 s'^2$$

$$\Rightarrow T_p^2 = T_0^2 + W^2\left(\frac{T_0^2}{W^2}\sinh^2\frac{Wx}{T_0}\right)$$

$$\Rightarrow T_p^2 = T_0^2 + T_0^2\sinh^2\frac{Wx}{T_0} = T_0^2\cosh^2\frac{Wx}{T_0}$$

$$\Rightarrow T_p = T_0\cosh\frac{Wx}{T_0}$$

When x increases then T_p increases for a given va of T_0. The minimum tension in the line is

The sag S can be found by putting $x = l/2$ in eqn

we get,

$$S = \frac{T_0}{W}\left[\cosh\left(\frac{Wl}{2T_0}\right) - 1\right]$$

$$\Rightarrow S = \frac{Wl^2}{8T_0}\left[1 + \frac{l^2}{48}\cdot\left(\frac{W}{T_0}\right)^2 + \cdots\cdots\right]$$

The Length of half line can be found by Putting $x = l/2$ in eqn ④ we get,

$$s' = \frac{T_0}{W}\sinh\frac{Wx}{T_0}$$

$$\Rightarrow \frac{Z}{2} = \frac{T_0}{W}\sinh\left(\frac{WL}{2T_0}\right)$$

$$\Rightarrow Z = \frac{2T_0}{W}\sinh\left(\frac{WL}{2T_0}\right)$$

$$\Rightarrow Z = l\left[1+\left(\frac{l^2}{24}\right)\left(\frac{W}{T_0}\right)^2+\left(\frac{l^4}{1920}\right)\left(\frac{W}{T_0}\right)^4+\cdots\right]$$

Conductor Vibration :-

Vibrations of Overhead line classified into two groups. (i) Resonant vibration (or Aeolian Vibrating)
(ii) Dancing or galloping.

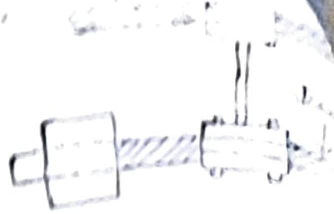
(i) Resonant Vibration :-

→ They are caused by Vortex phenomenon in Light winds (6- 20 $\frac{Km}{h}$).

→ These vibrations have low amplitudes (upto 5mm) and high frequencies (50-100 Hz)

→ Frequency of resonant vibration is $f = \frac{v}{d}$
where v is wind velocity in Km/h
and d is diameter of conductor in mm.

→ The Length of loop (half wave length) $= \frac{1}{2f}\sqrt{\frac{T}{W}}$
where T is tension in N and
W is conductor weight in $\frac{kg\text{-}wt}{m}$.



To minimise these vibrations, a stock bridge damper is used. It consists of two masses at the end of a short length stranded steel cable suspended from the conductor where the amplitud Vibration woul'd be maximum. The vibration movement of the damper and energy is abs by the inter strand friction in steel cab

→ A typical damper for use on 132 kV lin is about 60 cm long and weighs about

(ii) __Galloping or Dancing conductors :-__

→ They are caused by asymmetrical layer, ice formation.

→ These vibrations have high amplitude (up to and low frequency (1 to 2Hz).

✓

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Problem - ① An overhead line has a span of 336 m. The line is supported at a water crossing, from two towers whose heights are 33.6 m and 29 m above water level. The weight of conductor is 8.33 N/m and tension in the conductor is not to exceed 3.34×10^4 N. Find (a) clearance between the lowest point on the conductor and water (b) horizontal distance of this point from the lower support.

Solution:— Given, $L = \overset{336\,m}{\cancel{33.6}}\ M$, $W = 8.33\ N/m$, $T = 3.34 \times 10^4\ N$

Let x = distance of lowest point from support at low level

$h = 33.6 - 29 = 4.6\ m$

We know that, $x = \frac{L}{2} - \frac{Th}{WL}$

$$= \frac{\overset{336}{\cancel{33.6}}}{2} - \frac{3.34 \times 10^4 \times 4.6}{8.33 \times 336}$$

$$= 113.10\ m$$

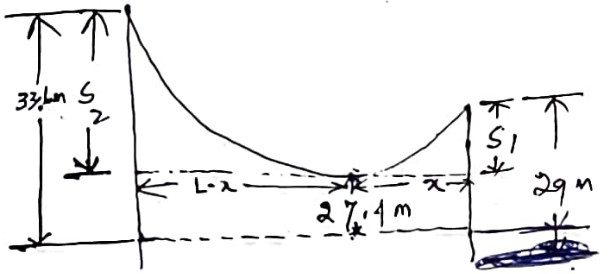
Sag at low level $= S_1 = \frac{Wx^2}{2T} = \frac{8.33 \times (113.10)^2}{2 \times 3.34 \times 10^4} = 1.5951\ m$

Sag at high level $= S_2 = \frac{W(L-x)^2}{2T} = \frac{8.33(336 - 113.10)^2}{2 \times 3.34 \times 10^4}$

$= 6.1956\ m$

(a) Clearance betn lowest point on the conductor and water $= 29 - 1.5951 = 27.40\ m$

(b) Horizontal distance of this point from lowest support $= x = 113.10\ m$.



Distribution ㉓

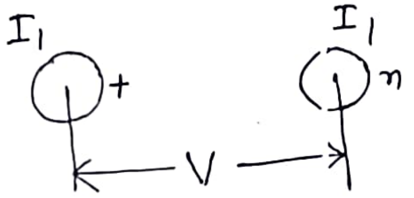
The Conductor system by means of which electrical energy is conveyed from bulk Power Sources (i.e generating stations or major substations supplied over transmission lines) to the consumers is known as distribution system.

Comparison of Various distribution System:-

When we compare the volumes of conductor material required in various distribution systems (i.e both DC and AC) then we consider the ~~ampo~~ amount of Power (P) transferred, the line length (l) and the line losses are same in each case.

Case(i). D.C two-wire system:-

In this system the neutral (n) acts as the return conductor and carries the same current as the positive conductor.



The resistance of ~~the~~ each conductor = R_1

Total line losses $= 2I_1^2 R_1 = 2\left(\frac{P}{V}\right)^2 R_1$ $\quad \left\{ \because I_1 = \frac{P}{V} = \text{line current} \right\}$

$$= \frac{2P^2}{V^2} \cdot R_1 \longrightarrow ①$$

Case(ii). D.C three wire system:-

In this system, the voltage from the two outer conductors to the neutral are opposite polarity. For a balanced system

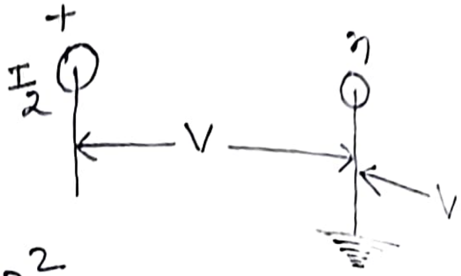
The current in the neutral conductor is
The losses in the neutral wire are zero
its volume can be neglected.

The resistance of each conductor = R_2

Total line losses = $2 \, \frac{I}{2}^{2} R_2$

$= 2\left(\frac{P}{2V}\right)^2 R_2$

$= \frac{P^2}{2V^2} R_2 \longrightarrow ② \left\{ \because \frac{I}{2} = \frac{P}{2V} = \text{line current} \right\}$



From eqn ① and ②, $\frac{2P^2}{V^2} R_1 = \frac{P^2}{2V^2} R_2$

$\Rightarrow R_2 = 4R_1$

$\Rightarrow \frac{R_1}{R_2} = \frac{1}{4}$ $\left\{ \because R \propto \frac{1}{A} \right\}$

$\Rightarrow \frac{A_2}{A_1} = \frac{1}{4}$

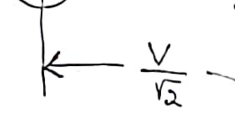
$\Rightarrow \frac{A_2 l}{A_1 l} = \frac{1}{4}$ $\left\{ \because \text{Volume} = V = Al \right\}$

$\Rightarrow \frac{V_2}{V_1} = \frac{1}{4}$

$\Rightarrow \frac{V_1}{V_2} = \frac{1}{1/4} \longrightarrow ③$

Case(iii) Single Phase AC System :-

In this system,
Peak voltage between conductors = V.
Rms voltage is $= \frac{V}{\sqrt{2}}$
Power factor = $\cos\phi$



$$\therefore \quad P = V_{rms} I_3 \cos\phi$$

$$\Rightarrow \quad P = \frac{V}{\sqrt{2}} I_3 \cos\phi$$

$$\Rightarrow \quad I_3 = \frac{P}{\frac{V}{\sqrt{2}} \cos\phi}$$

Resistance of each conductor $= R_3$

$$\therefore \text{ Total line losses } = 2 I_3^2 R_3 = 2\left(\frac{P}{\frac{V}{\sqrt{2}} \cos\phi}\right)^2 R_3 = \frac{4P^2}{V^2 \cos^2\phi} R_3 \longrightarrow ④$$

From equation ① and ④

$$\frac{2P^2}{V^2} R_1 = \frac{4p^2}{V^2 \cos^2\phi} R_3$$

$$\Rightarrow \quad R_1 = \frac{2}{\cos^2\phi} R_3$$

$$\Rightarrow \quad \frac{R_1}{R_3} = \frac{2}{\cos^2\phi}$$

$$\Rightarrow \quad \frac{A_3}{A_1} = \frac{2}{\cos^2\phi}$$

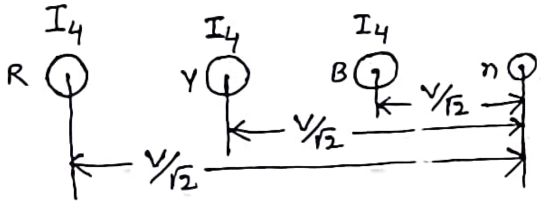
$$\Rightarrow \quad \frac{A_3 l}{A_1 l} = \frac{2}{\cos^2\phi}$$

$$\Rightarrow \quad \frac{V_3}{V_1} = \frac{2}{\cos^2\phi}$$

$$\Rightarrow \quad \frac{V_1}{V_3} = \frac{1}{2/\cos^2\phi} \longrightarrow ⑤$$

Case (iv) **3-phase 4-wire AC :-**

In this system the voltage between each phase conductor and neutral is $V/\sqrt{2}$.



For a balanced system, the current in ne
conductor is zero. The losses in neutr
wire are zero and its volume can be

The resistance of each conductor = R_4

Total line losses $= 3 I_4^2 R_4$

$$= 3\left(\frac{P}{3\left(\frac{V}{\sqrt{2}}\right)\cos\phi}\right)^2 R_4$$

$$= \frac{2}{3}\frac{P^2}{V^2\cos^2\phi} R_4 \longrightarrow ⑥$$

$\therefore I_4 =$

$= \frac{}{3(}$

From eqn ① and ⑥, $\frac{2P^2}{V^2} R_1 = \frac{2}{3}\frac{P^2}{V^2\cos^2\phi} R_4$

$$\Rightarrow R_1 = \frac{R_4}{3\cos^2\phi}$$

$$\Rightarrow \frac{R_1}{R_4} = \frac{1}{3\cos^2\phi}$$

$$\Rightarrow \frac{A_4}{A_1} = \frac{1}{3\cos^2\phi}$$

$$\Rightarrow \frac{A_4 l}{A_1 l} = \frac{1}{3\cos^2\phi}$$

$$\Rightarrow \frac{V_4}{V_1} = \frac{1}{3\cos^2\phi}$$

$$\Rightarrow \frac{V_1}{V_4} = 3\cos^2\phi \longrightarrow ⑦$$

But $V_1 = 2 A_1 l$

$V_2 = 2 A_2 l$

$V_3 = 2 A_3 l$

$V_4 = 3 A_4 l$

$$\therefore \frac{V_1}{V_4} = \frac{2 A_1 l}{3 A_4 l} = \frac{2}{3}(3\cos^2\phi)$$

$$\Rightarrow \frac{V_1}{V_4} = 2\cos^2\phi$$

$\because \frac{A_1 l}{A_4 l} = 3\cos^2$

$$\Rightarrow \frac{V_1}{V_4} = \frac{1}{1/2\cos^2\phi} \longrightarrow ⑧$$

From eqn ③, ⑤ and ⑧ we get

$$\boxed{V_1 : V_2 : V_3 : V_4 = 1 : \frac{1}{4} : \frac{2}{\cos^2\phi} : \frac{1}{2\cos^2\phi}} \rightarrow ⑨$$

From equation ⑨ it is clear that the three phase four wire system requires lesser conductor material than the single phase ac system.

Neutral wire of half cross-section :-

If the volume of neutral wire is considered and the neutral wire has half the cross-section of phase conductors in three phase four wire system then

$$\text{volume } V_5 = 3A_4 l + \frac{A_4}{2} l. \quad \left\{ \text{where } \frac{A_4}{2} = \text{area of cross-section of neutral wire} \right\}$$

$$= \frac{7A_4 l}{2}$$

$$= 3.5\, A_4 l.$$

$$\therefore \frac{V_1}{V_5} = \frac{2A_1 l}{\frac{7A_4 l}{2}} = \frac{4A_1 l}{7A_4 l} = \frac{4}{7}(3\cos^2\phi) \quad \left\{ \text{where as } \frac{A_1 l}{A_4 l} = 3\cos^2\phi \right\}$$

$$\Rightarrow \frac{V_1}{V_5} = \frac{12}{7}\cos^2\phi = \frac{1}{7/12\cos^2\phi} \rightarrow ⑩$$

From eqn ⑨ and ⑩ we get

$$\boxed{V_1 : V_2 : V_3 : V_4 : V_5 = 1 : \frac{1}{4} : \frac{2}{\cos^2\phi} : \frac{1}{2\cos^2\phi} : \frac{7}{12\cos^2\phi}}$$

A.C $3-\phi$, 4-wire Distribution :-

(i) Primary Distribution System :-

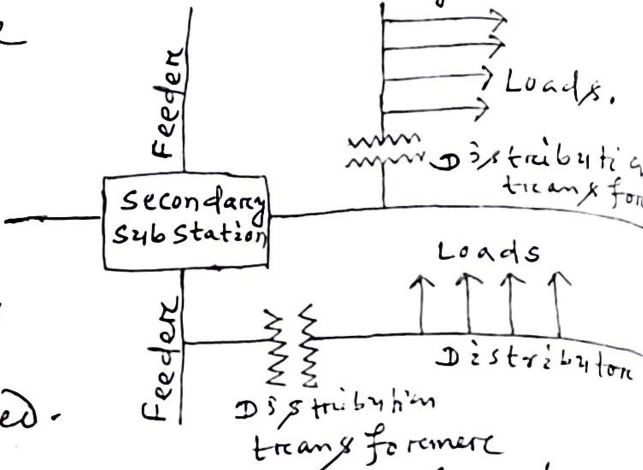
→ It is a distribution system which oper
at voltages such as 3.3, 6.6, 11 KV or hig

→ Electric Power from the generating sta
is transmitted through extra high tensi
transmission lines at voltages from 33 KV t
to various substations located in or near
the city. These substations are called
Secondary substations. At these substatio
the voltage in stepped down to 11 KV, 6.6 KV

→ The Primary distribution system consists of
radial feeders, Parallel feeders, ring Feede
or a network.

Radial Feeder :- It is the simplest, economi
and most commonly used i

It derives its name from the fact that the feeder radiates from the secondary substations and branches into sub feeders and laterals which extend into all parts of the area served.



The distribution transformers (11 KV/415V)
Connected to the Primary feeders, Sub feeder,
and laterals usually through fuse cut out

It is used to supply small and medium residental, commercial, industrial and non-critical loads.

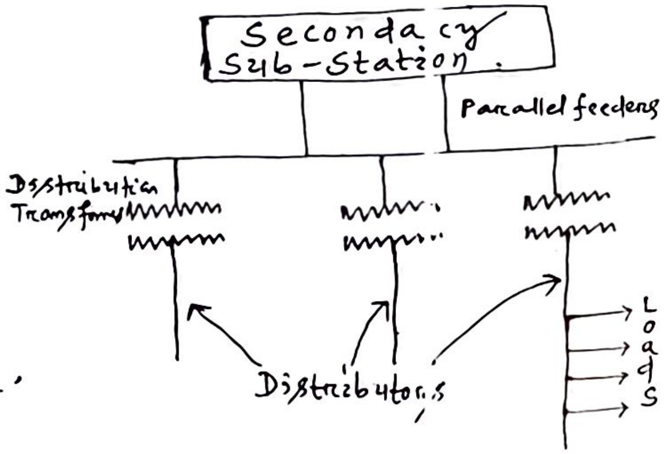
Radial feeder has the following drawbacks;

(i) When a fault occurs at any point on the feeder, supply to all consumers beyond the fault point towards the tail end gets interrupted.

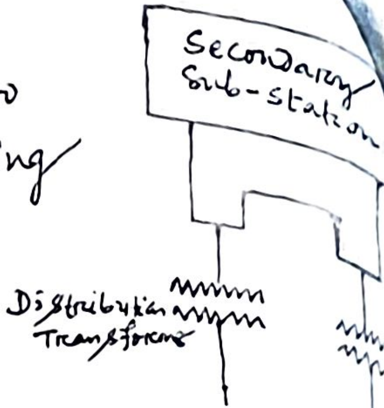
(ii) In case of increase in load demand, the length of the feeder has to be extended and it may result in a greater voltage drop. It may cause the voltage towards the tail end to reach a value below the permissible value.

Parallel feeders :- It consists of two radial feeders originating from the same or different secondary substations are run in parallel. Each feeder shares the total load equally in normal conditions. But each feeder has a capability to supply the entire load.

This system is expensive but reliability is increased as in case of fault on one feeder, the total load can be supplied by the healthy feeder.



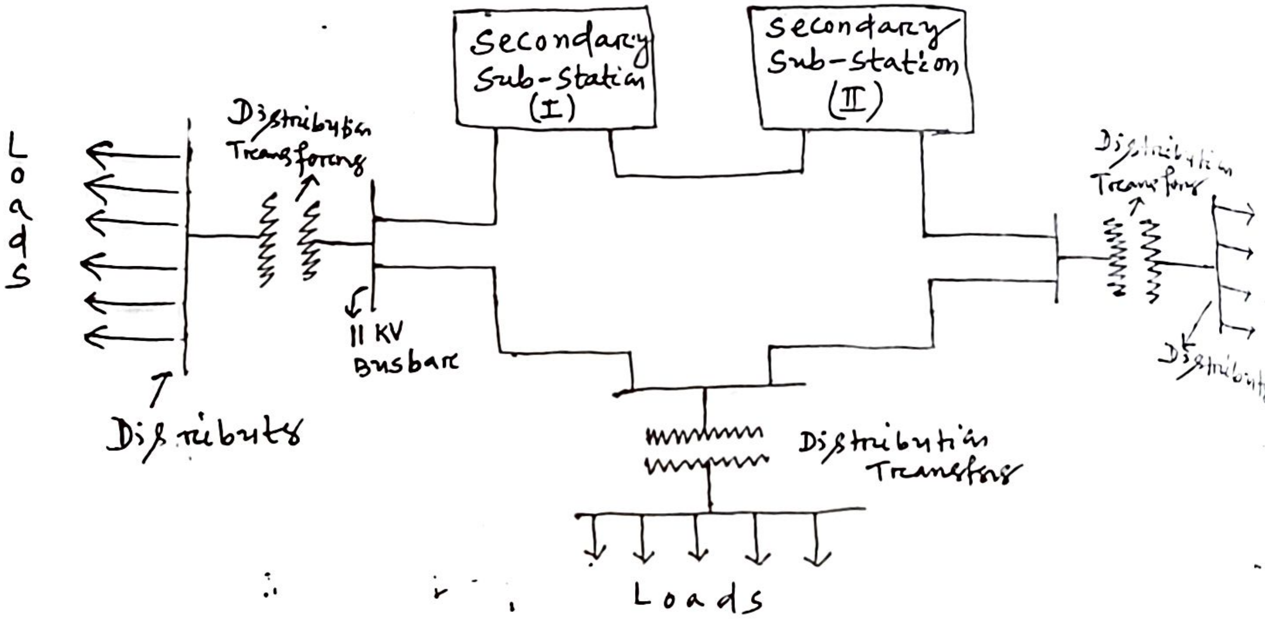
loop feeder :- A system of two or more radial feeders originating from the same or different secondary substations and separately routed through the load areas is known as loop feeder system.



If the ends of the two feeders are tied together through normally open switching devices then it is known as open loop system. If the ends are tied together by closed switching devices then it is known ring feeder.

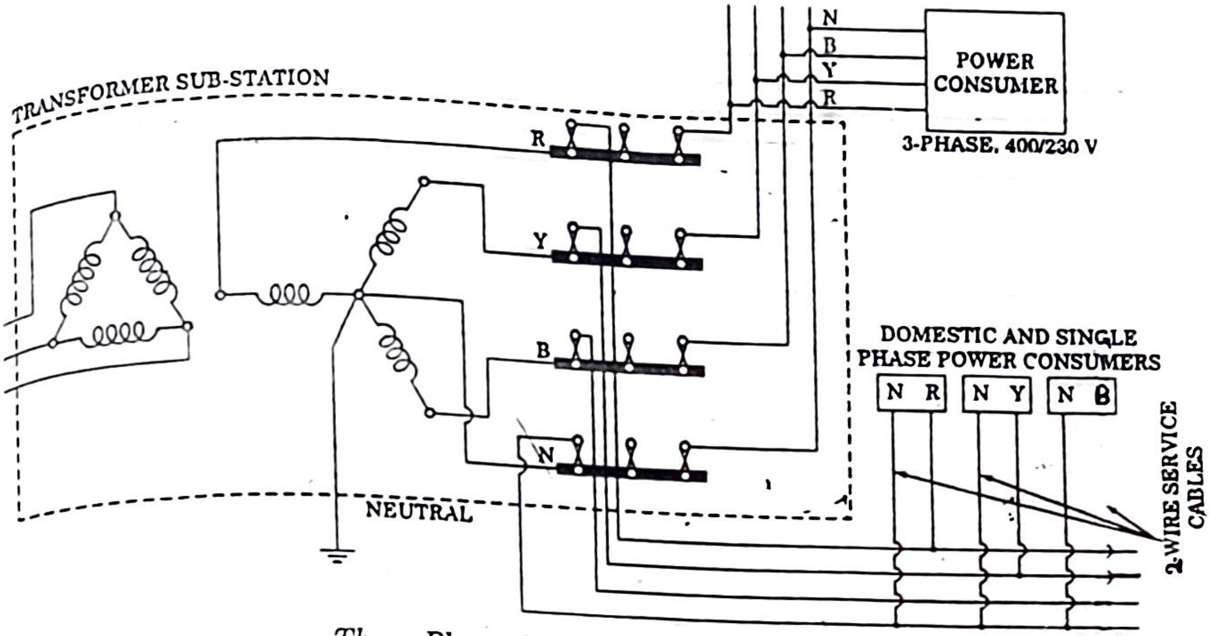
This system is most reliable for continuity of supply and gives better voltage regulation and less power losses

Inter-connected Network :-



When the feeder ring is energized from two or more than two generating stations or substations it is called inter-connected network system. Because of inter-connected feeders, Power can be supplied to all the distribution transformers. Such a system provides better reliability and flexibility and is employed in large metropolitan cities where continuity of supply is the most important.

(ii) Secondary Distribution System :-



Three-Phase, Four-Wire Distribution

(The secondary distribution system is shown in the figure)

→ The secondary distribution system cons of 3-ϕ, 4-wire distributors laid along road

→ 3-ϕ, 4-wire distribution layout is shown in Transformer substations are built at load of the area where the load is to be suppli

→ The substation contains high voltage switchge and bus bars and low voltage fuses or

→ The supply to the primary is by HV or EHV fee cable from the generating station. The low bus bars are distinguished by colour mark red, yellow, blue (for phases) and black (for neut

→ The local 4-wire distributors are connected in p to the bus bars through the fuses.

→ Domestic and other low voltage consumers ar connected to the distributors by 2-wire service cables which are tee jointed to distributors. In order to balance the three the consecutive services are connected to the different phases and neutral.

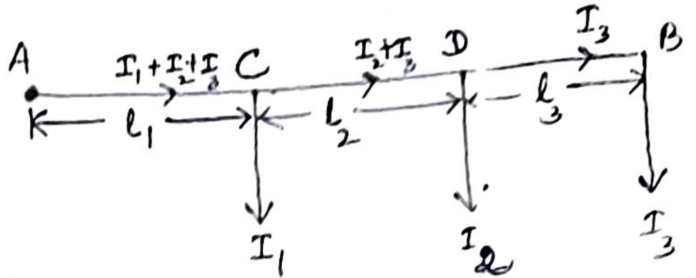
Voltage drop in DC Distributors :-

① Distributor Fed at one End :-

From fig.

A is the feeding point.

I_1, I_2, I_3 are the load currents tapped at different points.



r = resistance per unit length of the distributor (go and return)

∴ Resistance across AC = $l_1 r$
Resistance across CD = $l_2 r$
Resistance across DB = $l_3 r$.

$$\text{Voltage drop} = (I_1+I_2+I_3)l_1 r + (I_2+I_3)l_2 r + I_3(l_3 r)$$

$$= I_1 l_1 r + I_2(l_1+l_2)r + I_3(l_1+l_2+l_3)r$$

= Moment of Current I_1 about A + Moment of Current I_2 about A + Moment of Current I_3 about A

Thus total voltage drop at the far end of the distributor is the sum of the moments of various currents tapped off about the feeding point.

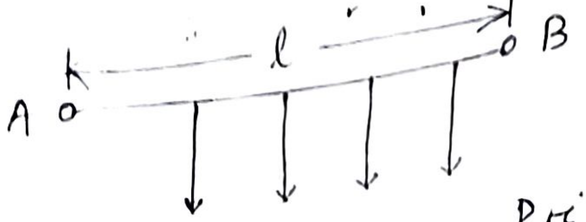
$l_1 r$ is the resistance of the distributor upto point where I_1 is tapped off, $(l_1+l_2)r$ is the resistance upto the point where I_2 is tapped off and so on.

② Uniformly Distributed load :-

Let us consider a distributor AB of length 'l' as shown in fig.

r = resistance per unit length of the distributor (go & return)

i = Current per unit length of the distributor.



A is the feeding Point.

Consider a Point 'C' from Point A.

The Current at Point $C = il - ix$

Consider an elementary length dx near Point 'C'.

Resistance of the elementary length $= r \cdot dx$

Voltage drop in this elementary length $= (il - ix) r$

Total voltage drop upto Point 'C' is,

$$V = \int_0^x (il - ix) r \cdot dx = irlx - \frac{irx^2}{2}$$

Voltage drop upto Point B can be determined by substituting $x = l$ in the above eqn.

$\therefore$ Voltage drop upto Point $B = irl \cdot l - \frac{irl^2}{2}$

$$= \frac{irl^2}{2} = \frac{(il)(rl)}{2}$$

$$= \frac{1}{2} IR$$

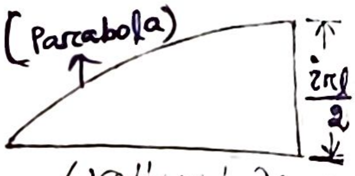
Where $I = il$ = total current fed from

$R = rl$ = total resistance of the distributor AB

From the above it is clear that in Uniformly loaded distributor total voltage drop is equal to that caused by the whole of the load assumed concentrat at the middle point.

The current loading and voltage loading diagram is shown.

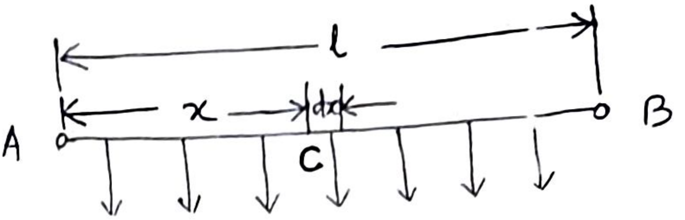
(current loading)



(voltage loading)

DC Distributor Fed from both ends at equal voltages:

et us consider
distributor AB
f length l as shown
n fig.



= resistance per unit length of the distributor
= current per unit length of the distributor.

'A' and 'B' are two feeding points.

V = voltage at 'A' = Voltage at B.

et 'C' being a point at a distance x from point 'A'.

Total current on conductor AB = il

Current supplied from each end = $\frac{il}{2}$

The current at point 'C' = $\frac{il}{2} - ix$

consider an elementary length dx near point 'C'.

Resistance of elementary length = $r \cdot dx$

Voltage drop of elementary length = $\left(\frac{il}{2} - ix\right) r \cdot dx$

Voltage drop in section AC = $\int_0^x \left(\frac{il}{2} - ix\right) r \cdot dx$

$$= \frac{ilrx}{2} - \frac{irx^2}{2}$$

Maximum voltage drop will occur at mid-point
i.e $x = l/2$.

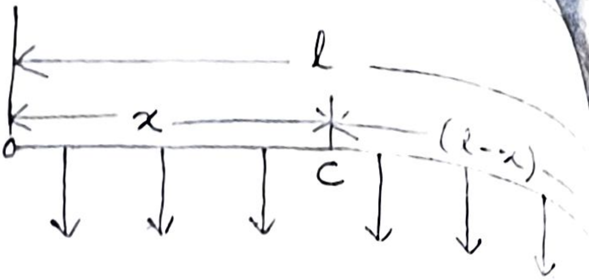
$$\therefore V_{max} = \frac{ilr \cdot \frac{l}{2}}{2} - \frac{ir\left(\frac{l}{2}\right)^2}{2} = \frac{irl^2}{8} = \frac{(il)(rl)}{8} = \frac{IR}{8}$$

Where $I = il$ = total current fed to the distributor.
$R = rl$ = total resistance of the distributor

Minimum Voltage occurring at mid point, $V_{min} = V - \frac{irl^2}{8} = V - \frac{IR}{8}$

(4) DC Distributor fed from both ends at differe

Consider a distributor AB of length 'l' as shown in fig.



r = resistance per unit length of the distributor.

$\dot{i}$ = current per unit length of the distributor.

'A' and 'B' are two feeding points.

V_A = Voltage at 'A'

V_B = Voltage at 'B'.

Let 'C' is a point at a distance 'x' from point and at the minimum voltage. So there is no current at point 'C'.

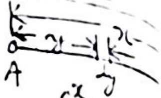
Current fed from feeding point 'A' = Current in AC

$= \dot{i}x$

Voltage drop in section AC $= \frac{\dot{i}rx^2}{2}$

Similarly Voltage drop in section BC $= \frac{\dot{i}r(l-x)^2}{2}$

Note



$$\int_0^x \dot{i}y \cdot r \cdot dy = \frac{\dot{i}r x^2}{2}$$

Now voltage at 'C' = Voltage at A − Voltage drop in AC

$$\therefore = V_A - \frac{\dot{i}rx^2}{2}$$

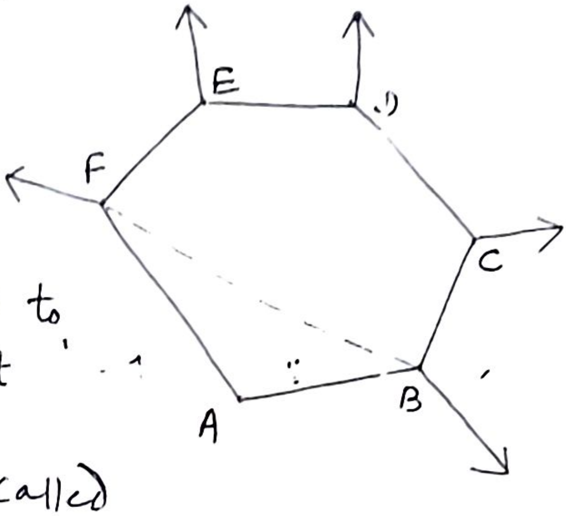
Also voltage at 'C' = Voltage at B − Voltage drop in BC

$$= V_B - \frac{\dot{i}r(l-x)^2}{2}$$

$$\therefore \quad V_A - \frac{\dot{i}rx^2}{2} = V_B - \frac{\dot{i}r(l-x)^2}{2}$$

$$\Rightarrow \boxed{x = \frac{V_A - V_B}{\dot{i}rl} + \frac{l}{2}}$$

b) Ring main Distributor :—



- Ring main distributor is shown in the fig.
- Here A is the feeding Point.
- A distributor arranged to form a closed circuit and fed at one or more than one Points is called ring main distributor.
- Such arrangement are better greater reliability and flexibility.
- If fault occurs at any section, it can be isolated from both sides and the continuity of supply is maintained for the remaining sections. The result is that only faulty section suffers discontinuity of supply.
- The ring distributor fed at one Point, can be considered equivalent to a straight distributor fed from both ends at equal voltages.
- Sometimes two or more ends of a ring main are connected through interconnector shown dotted in figure so as to form a complete network. The addition of an interconnector to a ring main reduces the voltage drop and Power loss in the system.

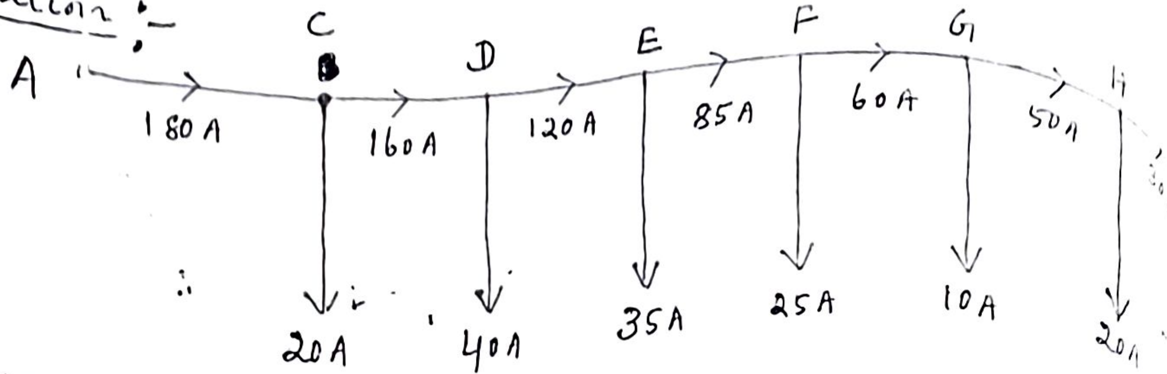
10 ① B R Gupta No = 16.5 (38)

A dc distributor AB is 500m long and is at Point A is 220V. Concentrated loads are from the distributor as under:

Distance from A in metre,	150	200	280	320	390
Current in Ampere,	20	40	35	25	10

The total resistance of double run is 0.2 ohm. Potential of Point B.

Solution :-



Given

AC = 150 m
CD = 50 m
DE = 80 m
EF = 40 m
FG = 70 m
GH = 60 m
HB = 50 m

resistance per unit length

$\therefore R_{AC} = 150 \times 4 \times 10^{-4} = 0.06\,\Omega$

$R_{CD} = 50 \times 4 \times 10^{-4} = 0.02\,\Omega$

$R_{DE} = 80 \times 4 \times 10^{-4} = 0.032\,\Omega$

$R_{EF} = 40 \times 4 \times 10^{-4} = 0.016\,\Omega$

$R_{FG} = 70 \times 4 \times 10^{-4} = 0.028\,\Omega$

$R_{GH} = 60 \times 4 \times 10^{-4} = 0.024\,\Omega$

$R_{HB} = 50 \times 4 \times 10^{-4} = 0.02\,\Omega$

Voltage at C = Voltage at A − Voltage drop in section

$V_C = V_A - 180(0.06) = 220 - 180(0.06) =$

Similarity $V_D = V_C - 160 \times 0.02 = 209.2 - 160 \times 0.02 = 206\text{ V}$

$V_E = V_D - 120 \times 0.032 = 202.16\text{ V}$

$V_F = V_E - 85 \times 0.016 = 200.8\text{ V}$

$V_G = V_F - 60 \times 0.028 = 199.12\text{ V}$

$V_H = V_G - 50 \times 0.024 = 197.92\text{ V}$

$V_B = V_H - 30 \times 0.02 = 197.32\text{ V}$

oblem-2 A dc ring main ABCDA is fed from point A from a 250 V supply and the resistances (including go & return) of various sections are as follows: AB = 0.02 Ω, BC = 0.018 Ω, CD = 0.025 Ω and DA = 0.02 Ω. The main supplies loads of 150 A at B; 300 A at C and 250 A at D. Determine the voltage at each load point.

If the points A and C are linked through an interconnector of resistance 0.02 Ω determine the new voltage at each load point.

lution :-

(i) Without interconnector :-

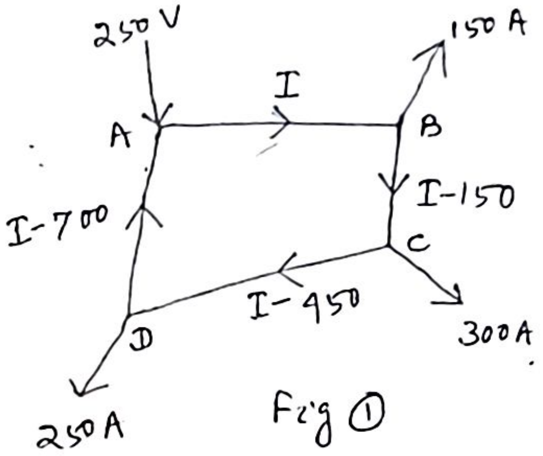


Fig ①

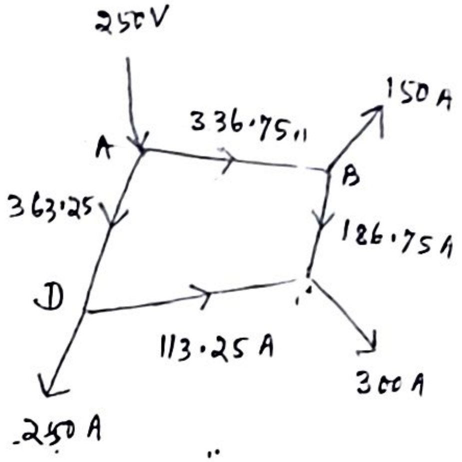


Fig ②

et I = current through section AB as shown in fig ①

According to KVL, $I_{AB} R_{AB} + I_{BC} R_{BC} + I_{CD} R_{CD} + I_{DA} R_{DA} = 0$

$\Rightarrow 0.02I + 0.018(I-150) + 0.025(I-450) + 0.02(I-700) = 0$

$\Rightarrow I = 336.75\ A$

e actual current distribution shown in fig ⑪.

voltage drop in AB = 336.75 × 0.02 = 6.735 V

" " " BC = 186.75 × 0.018 = 3.361 V

" " " CD = 113.25 × 0.025 = 2.831 V

" " " DA = 363.25 × 0.02 = 7.265 V

ltage at point B = 250 − 6.735 = 243.265 V

" " " C = 243.265 − 3.361 = 239.904 V

" " " D = 239.904 − (−2.831) = 242.735 V

(ii) With interconnector :-

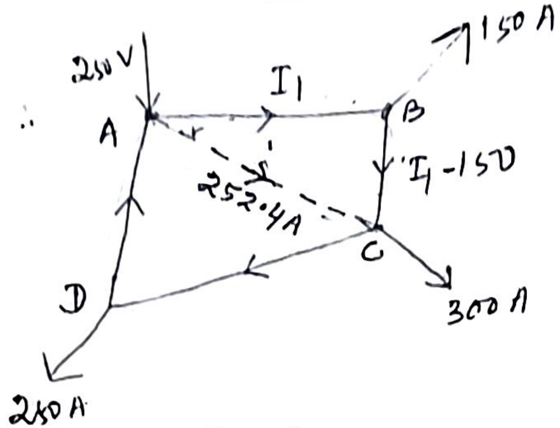


Fig ③

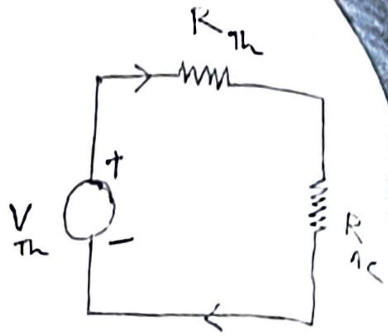


Fig ④

AC is the interconnector. The current in the interconnector can be found by applying Thevenin's theorem.

Voltage betn A and C = V_{Th} = Thevenin's voltage

∴ $V_{Th} = 250 - 239.904 = 10.096\ V$

R_{Th} = resistance viewed from points A and C

$$= \frac{(0.02+0.018)(0.02+0.025)}{(0.02+0.018)+(0.02+0.025)} = 0.02\ \Omega$$

But, $R_{AC} = 0.02\ \Omega$

Thevenin's equivalent ckt is shown in fig ④.

∴ Current in interconnector AC $= \frac{V_{Th}}{R_{AC}+R_{Th}} = \frac{10.096}{0.02+0.02}$

$= 252.4\ A$ from A to C

Let I_1 = current in section AB

∴ $I_1 - 150$ = " " " BC.

Applying KVL to ABCA, $0.02 I_1 + 0.018(I_1 - 150) - 0.02 \times 252.4$

$\Rightarrow I_1 = 203.15\ A$

The actual distribution of currents in the ring with interconnector is shown in fig ⑤

Drop in AB = 203.15 × 0.02 = 4.063 V
Drop in BC = 53.15 × 0.018 = 0.960 V
Drop in AD = 244.45 × 0.02 = 4.9 V

∴ Potential of B = 250 − 4.063 = 245.93 V
Potential of C = 245.93 − 0.96 = 244.97 V
Potential of D = 250 − 4.9 = 245.1 V

It is clear that the voltage drops in various sections of the distributor are reduced.

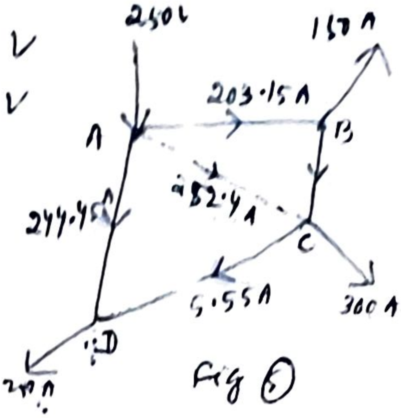
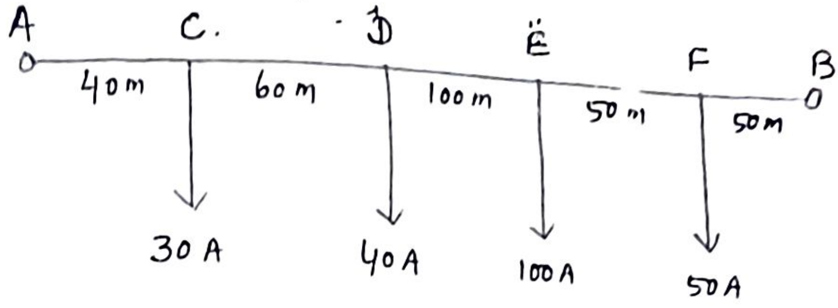
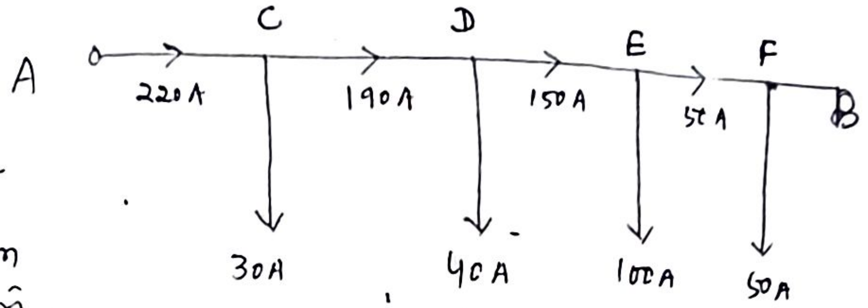


fig 6

2. Find the cross-sectional area of the distributor shown in fig. The distances are in meters. Take $\rho = 1.78 \times 10^{-8}$ Ω-m. The maximum voltage drop is not to exceed 10V. The conductor is fed from point A.



tion:-



r = resistance per unit length

al voltage drop in distributor is,

$$V = I_{AC} R_{AC} + I_{CD} R_{CD} + I_{DE} R_{DE} + I_{EF} R_{EF}$$

$$= 220 \times 40r + 190 \times 60r + 150 \times 100r + 50 \times 50r$$

$$= 37700\,r \text{ volts.}$$

But given $V = 10$ volts.

$$\therefore\ 37700r = 10$$

$$\Rightarrow r = \frac{10}{37700} = 2.652 \times 10^{-4}\ \Omega/m.$$

But $r = \rho \frac{l}{A}$

$$\Rightarrow A = \frac{\rho l}{r} = \frac{1.78 \times 10^{-8} \times 2}{2.652 \times 10^{-4}}$$

$\{ \because l = 2$ M (go and return) $\}$

As unit length $l = 1$

$$\Rightarrow A = 1.3423 \times 10^{-4}\ m^2$$

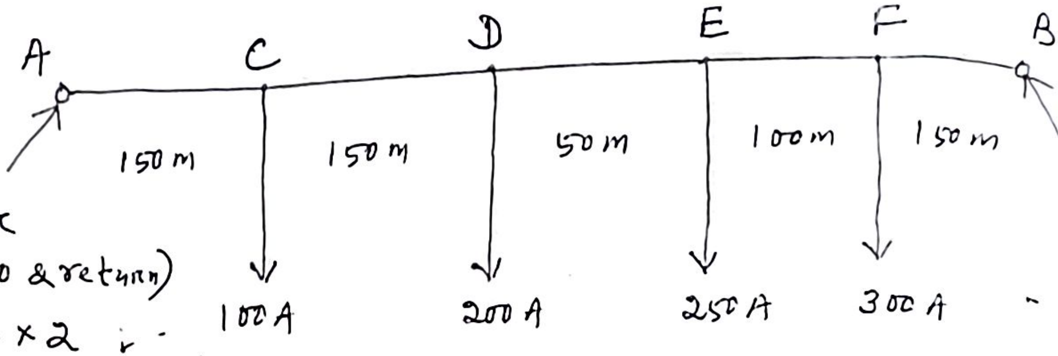
Pb-A

A dc two line distributor AB 600 m lon at 440 V from substation A and at 430 V fro substation B, the loads are

100 A at C, 150 m from A
200 A at D, 150 m from C
250 A at E, 50 m from D
300 A at F, 100 m from E

If each line conductor has a resistance of 0.01 Ω per 100 m then calculate the current supplied fro substations A and B and voltage across each loa

Solution:-



resistance per unit length (go & return)

$= r = \frac{0.01}{100} \times 2$

$= 2 \times 10^{-4}\ \Omega/m$

$R_{AC} = 150 \times 2 \times 10^{-4} = 0.03\ \Omega$

$R_{CD} = 150 \times 2 \times 10^{-4} = 0.03\ \Omega$

$R_{DE} = 50 \times 2 \times 10^{-4} = 0.01\ \Omega$

$R_{EF} = 100 \times 2 \times 10^{-4} = 0.02\ \Omega$

$R_{FB} = 150 \times 2 \times 10^{-4} = 0.03\ \Omega$

Let current through section AC = I

" " " CD = $I_A - 10$

" " " DE = $I_A - 3$

" " " EF = $I_A - 55$

" " " FB = $I_A - 8$

$\therefore\ V_B = V_A - I_{AC}R_{AC} - I_{CD}R_{CD} - I_{DE}R_{DE} - I_{EF}R_{EF} - I_{FB}R_{FB}$

$\Rightarrow 430 = 440 - I_A(0.03) - (I_A - 100)0.03 - (I_A - 300)0.01 - (I_A - 550)0.02$
$- (I_A - 850)0.03$

$\Rightarrow I_A = 437.5\ A$

Voltage drop in AC Distributors :-

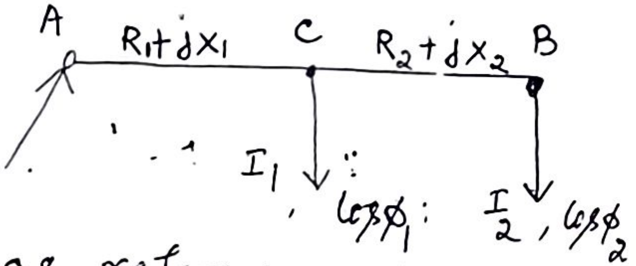
In dc system, the voltage drop is due to resistance. But in ac system the voltage drops ~~due~~ are due to combined effects of resistance, inductance and capacitance.

In a.c system power factor has to be taken into account. Loads tapped off ~~from~~ ~~from~~ from the distributor at different power factors.

Methods of Solving A.c distribution :-

(i) Power factors referred to receiving end :-

Let us consider an a.c distributor AB with concentrated loads I_1 and I_2 tapped off at points 'C' and 'B' as shown.



Take receiving end voltage V_B as reference vector.

R_1 and X_1 = resistance and reactance of section AC.

R_2 and X_2 = resistance and reactance of section CB.

$\cos\phi_1$ and $\cos\phi_2$ = lagging power factors at 'C' and 'B'.

R_1+jX_1 = impedance of section AC.

R_2+jX_2 = impedance of section CB.

Load current at point C $= I_1\angle{-\phi_1} = I_1(\cos\phi_1 - j\sin\phi_1)$

Load current at point B $= I_2\angle{-\phi_2} = I_2(\cos\phi_2 - j\sin\phi_2)$

Current in section CB $= I_2(\cos\phi_2 - j\sin\phi_2)$

Current in section AC $= I_1(\cos\phi_1 - j\sin\phi_1) + I_2(\cos\phi_2 - j\sin\phi_2)$

$= I_{AC}$

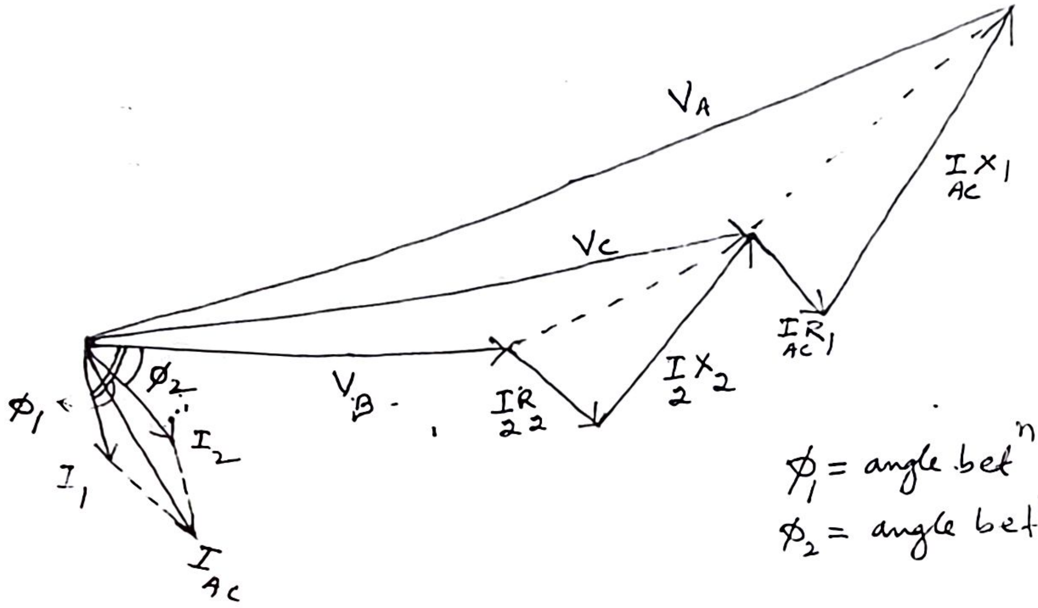
Voltage drop in section CB = $I_2(\cos\phi_2 - j\sin\phi_2)(R_2 + jX_2)$

Voltage drop in section AC = $\left[I_1(\cos\phi_1 - j\sin\phi_1) + I_2(\cos\phi_2 - j\sin\right.$

$[R_1 + jX_1] = \vec{V}_{AC}$

Sending end voltage = $\vec{V}_B + \vec{V}_{CB} + \vec{V}_{AC} = \vec{V}_S$

Sending end current = $\vec{I}_1 + \vec{I}_2 = \vec{I}_S$



ϕ_1 = angle betn V_B and I_1

ϕ_2 = angle betn V_B and I_2.

(ii) Power factors referred to respective load voltages

Let the power factors of loads are referred to their respective load voltages.

ϕ_1 = angle between V_C and I_1

ϕ_2 = angle between V_B and I_2

Voltage drop in section CB = $I_2(\cos\phi_2 - j\sin\phi_2)(R_2 + jX_2)$.

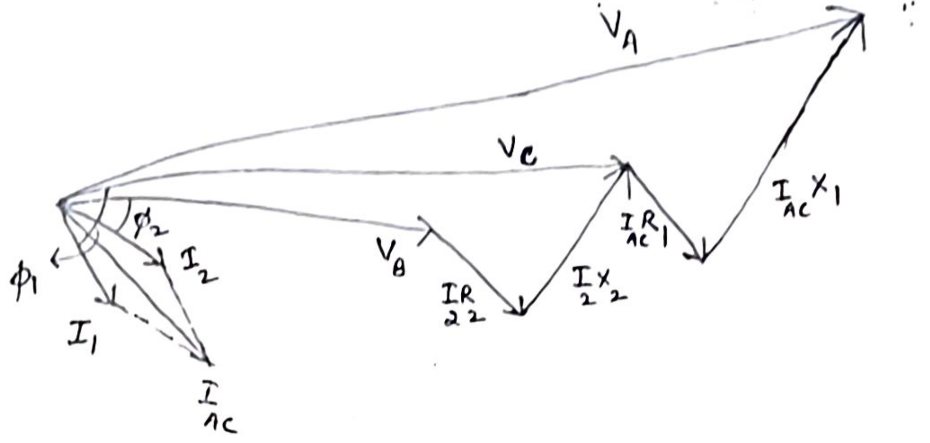
voltage at C = $\vec{V}_B$ + Drop in CB = $\vec{V}_C = V_C\angle\alpha$

$\vec{I}_1 = I_1\angle{-\phi_1}$ with respect to V_C

$\vec{I}_{AC} = \vec{I}_1 + \vec{I}_2$

Voltage drop in AC = $I_{AC}(R_1 + jX_1)$

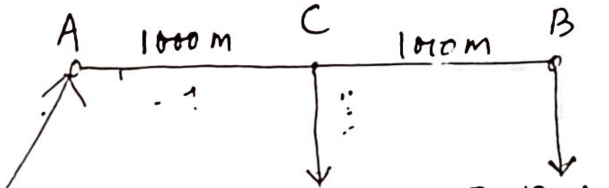
Voltage at Point A = $\vec{V}_B$ + Drop in CB + Drop in AC.



Problem :- A 1-φ distributor 2 km long supplies a load of 120 A at 0.8 p.f lagging at its far end and a load of 80 A at 0.9 p.f lagging at its mid-point. Both p.f are reffered to the voltage at the far end. The resistance and reactance per km (go and return) are 0.05 Ω and 0.1 Ω respectively. If the voltage at the far end is maintained at 230 V, Calculate
(i) sending end voltage
(ii) Phase angle between voltages at the two ends.

Solution :- AC = 1000 M = 1 KM

CB = 1000 M = 1 KM



$I_1 = 80A$

$\cos\phi_1 = 0.9$ (lag.

$\phi_1 = 25.8°$

$I_2 = 120 A$

$\cos\phi_2 = 0.8$ (la

$\phi_2 = 36.86°$

Impedance of AC = $(0.05 + j0.1)\,\Omega$

Impedance of CB = $(0.05 + j0.1)\,\Omega$

Given $V_B = 230$ Volts.

Let V_B be taken as reference vector.

$\therefore \vec{V}_B = V_B\angle 0 = 230\angle 0 = 230 + j0$.

(i) $\vec{I}_2$ = load current at B $= I_2 \underline{|-\phi_2} = 120 \underline{|-36.89}$

$= 96 - j72$

$\vec{I}_1$ - load current at C $= I_1 \underline{|-25.841} = 80 \underline{|-25.841}$

$= 72 - j34.87$

Current in CB $= \vec{I}_2 = (96 - j72)$ A

Current in AC $= \vec{I}_{AC} = \vec{I}_1 + \vec{I}_2 = 72 - j34.87 + 96 - j72$

$= (168 - j106.87)$ A

Voltage drop in CB $= (96 - j72)(0.05 + j0.1)$

$= (12 + j6)$ volts.

Voltage drop in AC $= (168 - j106.87)(0.05 + j0.1)$

$= (19.08 + j11.45)$ volts.

Sending end voltage $(\vec{V}_s) = \vec{V}_A = \vec{V}_B + \vec{V}_{CB} + \vec{V}_{AC}$

$= 230 + j0 + 12 + j6 + 19.08 + j11.45$

$= (261.08 + j17.45)$ volts.

$= 261.66 \underline{|3.82'}$ volts.

(ii) The Phase angle between voltages at two ends $= 3.82'$

(2) <u>Problem</u>. A 3-ϕ ring main ABCD fed at A at 11K supplies balanced loads of 50A at 0.8 P.f (lag) at B, 120A at unity P.f at C and 70A at 0.866 (lag) at D, the loads currents being referred to the supply voltage at A.

(48)

Impedance of section AB $= (1 + j0.6)\,\Omega$

" " " BC $= (1.2 + j0.9)\,\Omega$

" " " CD $= (0.8 + j0.5)\,\Omega$:

" " " DA $= (3 + j2)\,\Omega$

Calculate the currents in various sections and station bus-bar voltages at B, C and D.

Solution:-

elvin's law :- This law was stated by Lord Kelvin in 1881.

, The cost of a conductor is made up of two components.

(i) The interest on the capital cost of purchase and installation of conductor (plus an allowance for depreciation).

(ii) The cost of energy loss due to conductor resistance and in the case of cables, losses in metallic sheath and insulating material.

or a given length of the line, the weight and therefore the cost of conductor is proportional to the area of cross-section of the conductor. Hence the annual cost due to interest and depreciation is proportional to the cross-section.

Annual cost = Pa

where 'P' is a constant and 'a' is the area of cross-section

We know that $R \propto \frac{1}{a}$

For a given loading throughout the year, the energy loss is proportional to the resistance

i.e energy loss $\propto$ Resistance ~~resistance~~.

From the above, it is clear that

$$\text{Energy loss} \propto \frac{1}{a}$$

$$\Rightarrow \text{Energy loss} = Q \times \frac{1}{a}$$ (where Q is a constant)

$$\therefore \text{Total cost } (C) = Pa + \frac{Q}{a}.$$

For the total cost to be minimum $\frac{dc}{da} = 0.$

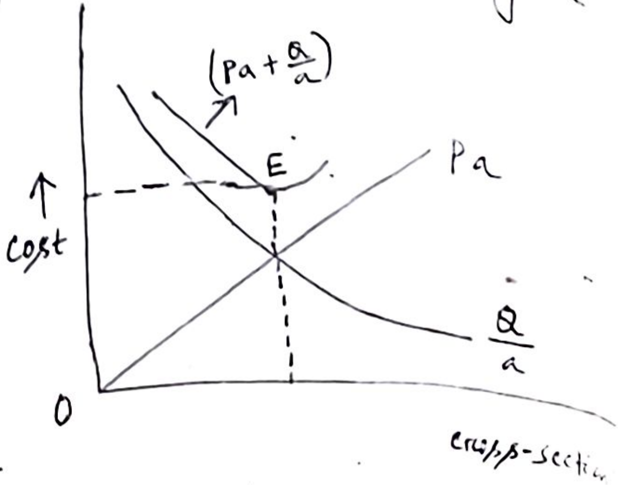
$$\Rightarrow \frac{dc}{da} = P - \frac{Q}{a^2} = 0$$

$$\Rightarrow a = \left(\frac{Q}{P}\right)^{0.5}$$

For this value of cross-section both the componants of the total cost (c) become equal, each being $\sqrt{PQ}$.
This is Kelvin's law.

→ **Statement** :- It states that the most economical cross-sect is that which makes the annual value of inte and depreciation of the Conductor equal t the annual cost of the energy wasted in th Conductor.

→ The graph of 'pa' is a straight line passing through the origin and the graph of $\frac{Q}{a}$ is a rectangular hyperbola



→ The total cost is minimum at the point where two graphs intersect each other.

→ Point E represents the most economical area of cross-section.

Limitations of Kelvin's law :-

→ It is not easy to estimate the energy loss in the line without actual load curves, which are not available at the time of estimation.

→ This law does not take into account several factors like safe current density, corona loss, mechanical strength etc.

(1.1)

blem: The cost of a 3-φ overhead transmission line is Rs (25000 a + 2500) per km, where a is area of cross-section of each conductor in cm². The line is supplying a load of 5 MW at 33 KV and 0.8 p.f lagging assumed to be constant throughout the year, energy costs 4 paisa per KWh and interest and depreciation total 10% per annum. Find the most economical size of the conductor. Given that specific resistance of conductor material is 10^{-6} Ω-cm.

olution: Resistance of each conductor

$$R = \rho\frac{l}{a} = \frac{10^{-6}\times 10^{5}}{a} = \frac{0.1}{a}\ \Omega$$

Line current, $I = \frac{P}{\sqrt{3}\,V\cos\phi} = \frac{5\times 10^{6}}{\sqrt{3}\times 33\times 10^{3}\times 0.}$ 109.35 A

Energy lost per annum $= \frac{3I^{2}Rt}{1000}$ kwh $= \frac{3(109.35)^{2}(0.1)(8760)}{1000\times a}$

$= \frac{31424}{a}$ kwh.

Annual cost of energy lost $= 0.04\times\frac{31424}{a} = \text{Rs } \frac{1256.96}{a}$

The capital cost (variable) of the cable is given to be Rs 25000 a per km length of the line;

∴ Variable annual charge = 10 % of capital cost (variable) of line.

= Rs 0.1 × 25000 a

= Rs 2500 a

According to Kelvin's law, for most economical cross-section of the conductor,

Variable annual charge = Annual cost of energy lost

⇒ 2500 a = 1256.96/a

⇒ a = 0.71 cm² ///

UNDERGROUND CABLE:-

UPTD 4/

→ An underground cable essentially consists of one or more conductors covered with suitable insulation and surrounded by a protective cover.

→ Although several type of cable are available, the type of cable to be used will depend upon the working voltage and service requirement. In general the cable must fulfill the

i) The conductor used in cable should be tinned stranded copper or aluminium of high conductivity.

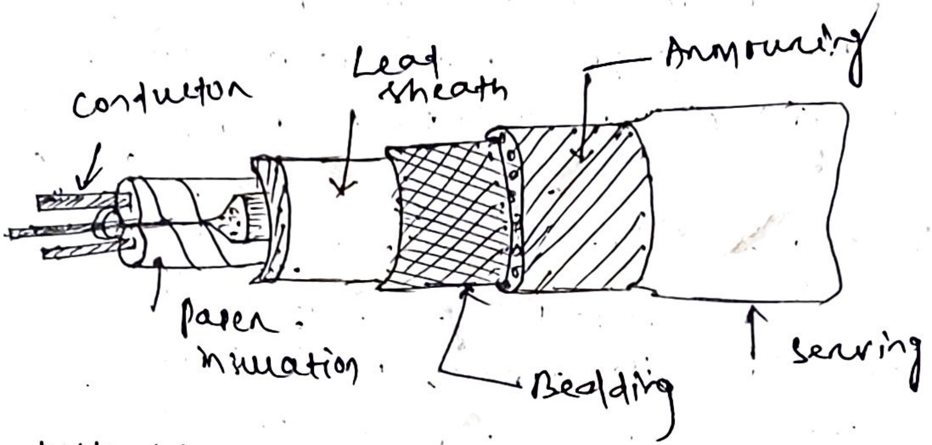
ii) Conductor size should be such that the cable carries the desired load current without overheating and causes voltage drop within permissible limits.

(iii) The cable must have proper thickness of insulation in order to give high degree of safety and reliability at the voltage for which it is designed.

(iv) The cable must have proper thickness of insulation in order to give high degree of safety.

(v) The cable must be provided high mechanical strength.

~~INSULATIONS~~



Core or conductors:-

→ A cable may have one or more than one core depending upon the type of service for which it is intended.

→ the above fig. have 3-conductor is for 3-ϕ service

→ The conductor are made of tinned copper and aluminium and are usually stranded in order to provide flexibility to the cable.

INSULATION!-

→ Each core or conductor is provided with a suitable thickness of insulation to isolate the conductor from each other and from the surrounding.

→ The thickness of wire depend upon the voltage to be withstood by the cable.

→ Insulating material should have high dielectric strength, high insulation resistance, good mechanical strength and should able to withstand temp from about -20°C to over 100°C.

→ Many insulating material have been developed and are used in cable manufacture.

- VIR cables
- Elastomer insulated cable
- PVC cable.
- Polythene insulated cable
- XLPE cable.
- Paper insulated cable

Sheath:-

→ In order to protect the cable from moisture, gases or other damaging liquids in the soil and atmosphere, a metallic sheath of lead or aluminium is provided over the insulation

→ Many years ago lead was established as a suitable material for sheathing. The main advantage of lead sheaths are they are flexibility, high corrosion resistance. Some disadvantage of lead are its large mass, low mechanical strength, small resistance to vibration

→ Aluminium is now being used as a sheathing material. Aluminium sheathing has a greater mechanical strength than lead sheath, low weight and low flexibility.

Bedding

Over the metallic sheath is applied a layer of bedding which consist of a fibrous material like jute or hessian tape. The purpose of bedding is to protect the metallic sheath against corrosion and from mechanical injury due to armouring.

Armouring

Over the bedding, armouring is provided which consist of one or two layer of galvanised steel wire or steel tape. Its purpose is to protect the cable from mechanical injury while laying it and during the course of handling. Armouring may not be done in the case of some cables.

Serving

In order to protect armouring from atmosphere conditions, a layer of fibrous material similar to bedding is provided over the armouring. This is known as serving.

CLASSIFICATION OF CABLES

→ The effective resistance Reff of each conductor of a cable has to be calculated by taking into account the dc resistance, skin effect, proximate effect, sheath loss effect and armour loss.

(a) Dc Resistance:—

* In calculating the dc resistance the increase in resistance due to temp, effect of stranding and laying should be taking into account.

* The conductor temp must not exceed 85°C for ~~[illegible]~~ pressurized cable. For other cable the specified temp. vary from 50°C to 75°C. The dc resistance at the maximum temp may be 20% more than that of 20°C.

* The resistance is multiplied by 1.02 to account for stranding and in case of multicore cables again multiplied by 1.02 to account for additional length of conductor resulting from the lay of strand and of core in manufacture.

(b) Skin effect:—

→ The increase in area due to skin effect depends mainly on the cross sectional area.

→ For 400 mm2 or 600 mm2 crosssectional area, the increase in resistance at 50 Hz is about 3.5% and 6.5% respectively.

(c) Proximate ~~loss~~ effect:—

The increase in resistance due to non-uniformity of current density, over the cross sectional ~~area~~ of the conductor, caused by magnetic field of current in other phase conductor is greater in single core cable than the three core cable for a given size.

→ ~~For~~ For three core belted cables the increase in resistance due to proximate effect is about the same magnitude as that due ~~to~~ to skin effect. The dc resistance at the operating temp as modified for stranding, laying, skineffect and proximate effect is termed as ac resistance i.e R_{ac}.

Sheath Loss:-

When alternating current flows in the core of a cable, the core and the sheath act as ~~a~~ the primary and secondary of air core transformer. An emf is induced in the sheath leading to flow of eddy currents and longitudinal circulating currents. This current gives rise to sheath losses. The losses due to eddy current in the sheath are usually negligible except in very large cable.

Armour Loss:-

→ These loss ~~due to~~ partly due to the eddy currents in the armour and partly due to hysteresis. The losses due to eddy current are of greater importance.

→ It is not a standard practice to provide a single core cable with armour of magnetic materials because of large armour losses and larger inductive reactance

→ In multi core cable the armour losses are generally negligible for conductor sections less than 200mm2 but may be high as 20%. of conductor reactance loss for some 3-core cables.

If sheath loss = λ_1 × conductor loss ~~[illegible]~~.
armour loss = λ_2 × conductor loss , then.

$$\boxed{R_{eff} = R_{ac}(1+\lambda_1+\lambda_2)}$$

CONDUCTOR INDUCTIVE RE

$$L = 2\times10^{-7} \log\left(\frac{D}{r'}\right) H/m$$

$$L = 2\times10^{-7} \log\left(\frac{\sqrt[3]{D_{12} D_{23} D_{31}}}{r'}\right) H/m.$$

→ above both the equation can be used to calculate the inductive reactance of each conductor of single core any 3-core cables respectively.

→ The results have to be modified to take into account the following effects:-

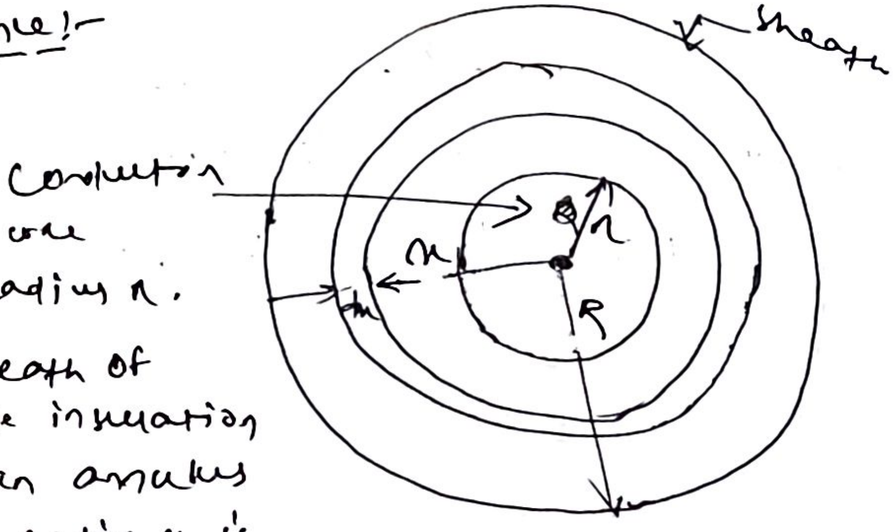
(a) Reduction due to mutual coupling with the sheath. This is generally small except in some cases such as large single core cable with aluminium sheath.

(b) Reduction due to the lower effective spacing between cables with conductor of cross section other than circular.

(c) Increase due to mutual coupling with armour of 3-core cables. this may be to an extent of 10 %.

PARAMETER OF SINGLE CORE CABLES:-

Insulation Resistance:-



→ Fig shows a single core cable of conductor radius r.

→ The cable has a sheath of inside radius R. The insulation resistance dR_{ins} of an annulus of thickness dx at radius x is

$$dR_{ins} = \frac{\rho\, dx}{2\pi x} \text{ ohm/m}$$

ρ = resistivity of insulating materials.

$$\Rightarrow R_{ins} = \int_r^R \frac{\rho\, dx}{2\pi x} = \frac{\rho}{2\pi} \ln x\Big]_r^R = \frac{\rho}{2\pi} \ln \frac{R}{r} \text{ ohm/m}$$

If the cable has a length of l meters.

Insulating resistance = $\frac{\rho}{2\pi l} \ln \frac{R}{r}$ ohm.

→ Avg value of ρ for impregnated paper varies from 5×10^{12} to 8×10^{12} ohm-mtr at 15°C

Change in resistivity of insulating material with temp is described by the equation.

~~$\rho_t = \rho$~~ $\boxed{\rho_t = \rho_0 e^{-\alpha t}}$

where ρ_t is resistivity at t°c, ρ_0 is resistivity at t_0°c.

α = constant.

Capacitance

→ Since single core cable has an earthed metallic sheath ⇒ there is an electric field between the conductor and sheath

→ Let the charge on the surface of conductor be q-coloumb per meter length of cable.

The Electric flux density D_r at a radius r is

$$D_r = \frac{q}{2\pi r} \quad C/m^2$$

Electric field intensity E_r at radius r

$$E_r = \frac{D_r}{\varepsilon_0 \varepsilon_r} = \frac{q}{2\pi \varepsilon_r \varepsilon_0 r} \quad V/m$$

ε_r = relative permittivity of the cable insulation and

$\varepsilon_0 = 8.85\times10^{-12}$ F/m.

The potential difference between the core and sheath is

$$V = \int_r^R E_r\, dr = \frac{q}{2\pi\varepsilon_0\varepsilon_r} \ln R/r \text{ volts.}$$

Capacitance between core and sheath is

$$C = \frac{q}{V} = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln R/r} \quad F/m.$$

Electric stresses :-

$$V = \int_r^R E_x\,dx = \int_r^R \frac{q}{2\pi\epsilon_0\epsilon_r x}\,dx = \frac{q}{2\pi\epsilon_0\epsilon_r}\ln\frac{R}{r}$$ ①

$$E_x = \frac{D_x}{\epsilon_r\epsilon_0} = \frac{q}{2\pi\epsilon_0\epsilon_r x} \Rightarrow \frac{q}{2\pi\epsilon_0\epsilon_r} = x E_x$$

$$\Rightarrow V = x E_x \ln R/r \Rightarrow \boxed{E_x = \frac{V}{x \ln R/r}}$$ — (A)

maximum potential gradient occurs at x=r i.e. at the surface of the conductor and is given by

$$\boxed{E_{max} = E_r = \frac{V}{r\ln R/r}}$$ — (B)

minimum potential gradient occurs at x=R

$$E_{min} = E_R = \frac{V}{R\ln R/r}$$ — (C)

$$\boxed{\frac{E_{max}}{E_{min}} = R/r}$$ — (D)

→ The above analysis assume that the conductor is perfectly circular. Stranding of conductor increases Emax by 20%

→ For a given value of V and R, there is a certain value of r which gives minimum value of Emax.

Emax is minimum when $(r \ln R/r)$ is maximum

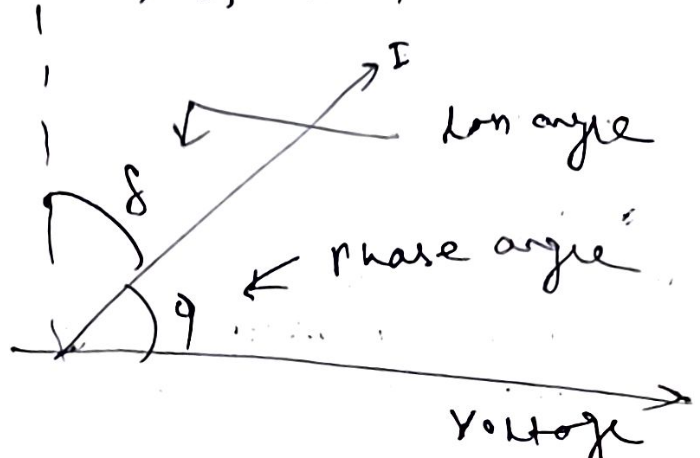
$$\frac{d}{dr}\left[r\ln R/r\right] = 0 \Rightarrow \boxed{R = 2.718\,r}$$

→ It may be noted that the variation of Emax with r is not large. When r changes from 0.5R to 0.25R, Emax changes by about 6%.

Dielectric Loss:-

→ A cable has a capacitance between core and sheath. When a voltage is applied to unloaded cable a capacitive current flows. Since the resistivity of the insulation is not infinite, leakage current flows and a power loss occurs.

→ With ac voltages the phenomenon of dielectric absorption also contribute to power loss. Thus a cable behaves as an imperfect capacitor and the ~~total~~ current, under unloaded condition, leads the voltage not by 90° but by an angle (90−δ) as shown



dielectric $P_d = VI\cos\phi = VI\cos(90-\delta)$

$= VI\sin\delta$

$= \omega C V^2 \sin\delta$

C → capacitance of cable, V = line to ground voltage.

GRADING OF CABLES:-

- The value of E_{max} has to be kept within limits depends on margin of safety and permissible degree of dielectric heating. Since the insulation away from the core is under stressed, there is an avoidable waste of insulation.

- Grading which reduce the amount of insulation by redistribution of stress so as to increase the stress in outer layer of insulation without increasing it at conductor surface.

→ Two Methods
  - capacitance grading
  - Inter sheath grading

Capacitance grading:-

- This method involves the use of two or more layer of dielectrics having different permittivities, those with higher permittivities being nearer to the conductor.

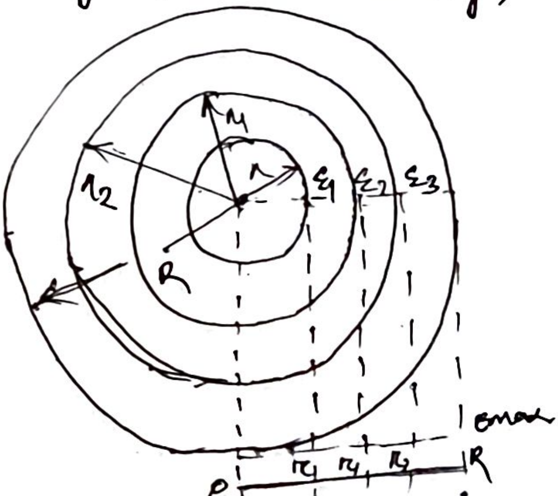
Electric field intensity = $E_r = \frac{D_r}{\varepsilon_0 \varepsilon_r} = \frac{q}{2\pi \varepsilon_0 \varepsilon_r r}$

→ If it were possible to vary permittivity with radius r such that

$$\varepsilon_r \propto \frac{1}{r} = \frac{m}{r}$$

$$E_r = \frac{q}{2\pi \varepsilon_0 \left(\frac{m}{r}\right) \times r} = \frac{q}{2\pi \varepsilon_0 m}.$$

Thus E_r is constant throughout the thickness of insulation. such a gradation is evidently, not possible)



from r = r to r = r1 the dielectric with relative permittivity ε_1 is used.

At $r = r$, $E = \dfrac{q}{2\pi\varepsilon_0\varepsilon_1 r}$

at $r = r_1$, $E = \dfrac{q}{2\pi\varepsilon_0\varepsilon_1 r_1}$

From $r = r_1$ to $r = r_2$, the dielectric with relative permittivity ε_2 is used

At $r = r_1 \Rightarrow E = \dfrac{q}{2\pi\varepsilon_0\varepsilon_2 r_1}$

$r = r_2$, $E = \dfrac{q}{2\pi\varepsilon_0\varepsilon_2 r_2}$

from $r = r_2$ to $r = R$ the dielectric with relative permittivity ε_3 is used

At $r = r_2$ $\quad E = \dfrac{q}{2\pi\varepsilon_0\varepsilon_3 r_2}$

$r = R$ $\quad E = \dfrac{q}{2\pi\varepsilon_0\varepsilon_3 R}$

if all the three dielectric operate at same maximum electric intensity, then

$$\frac{1}{\varepsilon_1 r} = \frac{1}{\varepsilon_2 r_1} = \frac{1}{\varepsilon_3 r_2}$$

$$\Rightarrow \varepsilon_1 r = \varepsilon_2 r_1 = \varepsilon_3 r_2$$

The operating voltage V is

$$V = \int_r^{r_1} E_r\,dr + \int_{r_1}^{r_2} E_r\,dr + \int_{r_2}^{R} E_r\,dr$$

$$= \int_r^{r_1} \frac{q}{2\pi\varepsilon_0\varepsilon_1 r}dr + \int_{r_1}^{r_2} \frac{q}{2\pi\varepsilon_0\varepsilon_2 r}dr + \int_{r_2}^{R} \frac{q}{2\pi\varepsilon_0\varepsilon_3 r}dr$$

$$= \frac{q}{2\pi\varepsilon_0\varepsilon_1}\ln\left(\frac{r_1}{r}\right) + \frac{q}{2\pi\varepsilon_0\varepsilon_2}\ln\left(\frac{r_2}{r_1}\right) + \frac{q}{2\pi\varepsilon_0\varepsilon_3}\ln\left(\frac{R}{r_2}\right)$$

~~$= E_{max}[r_2$~~

$$= \frac{q}{2\pi\varepsilon_0\varepsilon_1 r}\, r\ln\frac{r_1}{r} + \frac{q}{2\pi\varepsilon_0\varepsilon_2 r_1}\, r_1\ln\frac{r_2}{r_1} + \frac{q}{2\pi\varepsilon_0\varepsilon_3 r_2}\, r_2\ln\frac{R}{r_2}$$

$$= E_{max}\left[r\ln\frac{r_1}{r} + r_1\ln\frac{r_2}{r_1} + r_2\ln\frac{R}{r_2}\right]$$

Intersheath grading :-

→ In this method only one dielectric is used but the dielec is separated into two or more layers by thin metallic intersheaths, maintained at appropriate potentials by conne them to tapping on winding of transformer feeding the cab

→ There is a fixed voltage between the inner and outer radii of each sheath.

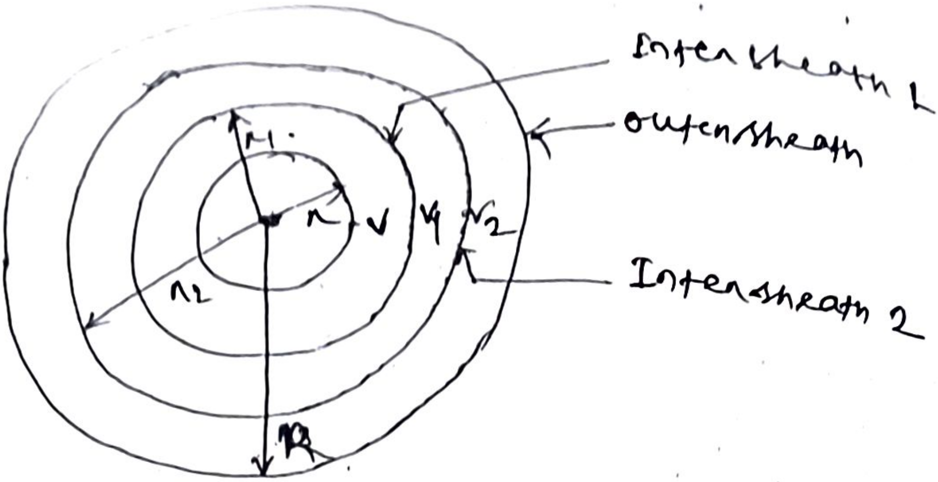


Fig shows a single core cable with two intersheaths. The different radii are r, r_1, r_2 and R. The potential difference between conductor and first intersheath is $V - V_1$, that between first and second intersheath is $V_1 - V_2$ and that between second intersheath and outer sheath is V_2.

$$E_{max1} = \frac{V - V_1}{r \ln\left(\frac{r_1}{r}\right)}$$

$$E_{max2} = \frac{V_1 - V_2}{r_1 \ln\left(\frac{r_2}{r_1}\right)}$$

$$E_{max3} = \frac{V_2}{r_2 \ln\left(R/r_2\right)}$$

If the value of maximum and minimum potential gradients in the three sections are kept the same, we get

$$\boxed{\frac{r_1}{r} = \frac{r_2}{r_1} = \frac{R}{r_2} = \alpha.}$$

$$\frac{V-V_1}{r\ln d} = \frac{V_1-V_2}{r_1\ln d} = \frac{V_2}{r_2\ln d}.$$

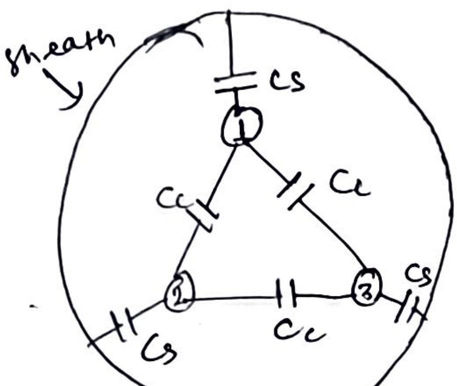
If the cable does not have any intersheath, the maximum voltage gradient is $E_{max} = \frac{V}{r\ln(R/r)}$

- Both the method for grading of cables involves practical difficulties. When capacitance grading the difficulty exists in obtaining different material having required ~~having required~~ values of permittivities. The use of rubber ($\varepsilon_r = 4-6$) and impregnated paper ($\varepsilon_r = 3-4$) have been suggested. Rubber is very expensive and the cost of dielectric becomes prohibitive.

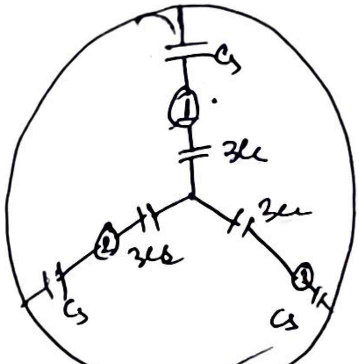
- With intersheath grading it may be difficult to arrange for proper voltage of intersheaths. The jointing of cable having intersheaths also poses problems. The charging current may cause overheating of intersheaths especially in long cables.

CAPACITANCE OF THREE CORE BELTED CABLE:-

- Three core belted cables are used only upto 11 kV. In this cable a potential difference exists between any two pairs of conductors and also between each conductor and sheath.
- Thus, there is an electric field between any two pairs of conductors and also between each conductor and sheath.
- Consequently there is capacitance, Cc between any two pair of conductors and capacitance Cs between each conductor and sheath.



Capacitance ~~of~~ in 3 core belted cable



(Equivalent capacitance)

→ The overall field pattern is very complicated and can be studied only in an electrolytic tank. The capacitance can also be more easily obtained by measurement.

→ The three delta connected capacitance C_c can be replaced by three star connected capacitance each value $3C_c$ using delta star transformation.

→ The capacitance to sheath can be assumed to be in parallel with star connected capacitance of each core to earthed neutral i.e $\boxed{C = 3C_c + C_s.}$ ✓

The capacitance C_c and C_s are obtained by the following measurement :

(a) Any two conductor are connected to sheath and the capacitance C_a between this combination and the third conductor is measured.

$$\boxed{C_a = 2C_c + C_s.}$$

(b) All the three conductors are joined together and capacitance C_b between combination and sheath is measured.

$$\boxed{C_b = 3C_s.}$$

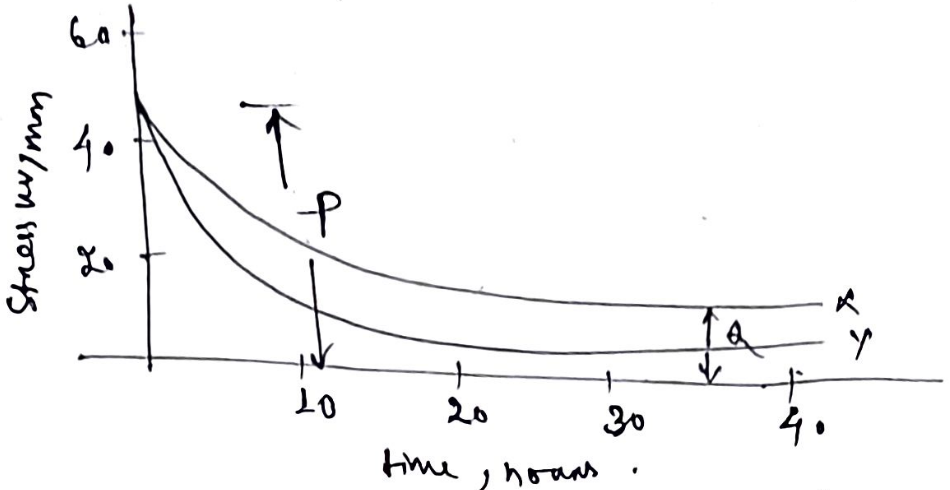
$$C_s = C_b/3, \quad C_c = \frac{C_a - C_s}{2} = \frac{C_a}{2} - \frac{C_s}{2} = \frac{C_a}{2} - \frac{C_b}{6}$$

$$\rightarrow \boxed{C_c = \frac{C_a}{2} - \frac{C_b}{6}.}$$

BREAKDOWN OF CABLES:-

Breakdown Characteristics:-

The electric stress which can cause the breakdown of the insulating material of a cable is not constant but depends on the time for which the stress is applied.



- Fig shows the electrical stress vs time to breakdown curve for two dielectrics. Curve x is for a well impregnated homogenous paper dielectric. P is is the instantaneous breakdown strength for dielectric and Q is the long term breakdown strength.
- The long term breakdown reach about 10 year. Curve y is for poor impregnated paper.
- Moisture and air filled voids decreases the breakdown strength and lead to cable breakdown.

Types of breakdown:-

(a) Puncture Breakdown:-

In this breakdown a discharge between the conductor and the sheath at a weak point in dielectric and a hole is formed in the dielectric. A defect in the dielectric which may lead to puncture. ~~is indicated in the initial laboratory test on a specimen of~~..

(b) Thermal instability:-

→ The charging current of a cable leads the voltage by an angle $(90-\delta)$ where δ = loss angle of the dielectric.

→ The dielectric loss and the ohmic loss in the conductor heat the insulation and raise its temp.

After a certain temp. is reached, a further increase in temp increases the loss angle which causes an increased power dissipation in the dielectric which increases the temp even further.

- The process becomes cumulative unless the rate of heat dissipation becomes equal to or greater than rate of heat generation. If the process becomes cumulative the dielectric breakdown may occur.

✓ Modern cable dielectric have very small loss angles and thermal instability doesnot occur under normal operating condition.

(c) Tracking Following void Formation:-

System operating problems with underground cable :-

(a) **Charging current :-** A cable has high capacitance which results in charging current and reactive power, If V is line to line voltage, the charging current I_c is

$$I_c = \omega C V / \sqrt{3} = 2\pi f C \times \frac{V}{\sqrt{3}}$$

3-phase reactive power $\sqrt{3}\, V I_c$

$$= \sqrt{3}\, V \left(2\pi f \frac{C V}{\sqrt{3}} \right).$$

$$= \cancel{x}^2$$

$$= \sqrt{3}\, V \left(\frac{2\pi f V}{\sqrt{3}} \right) \left(\frac{2\pi \varepsilon_0 \varepsilon_r}{\ln (R/r)} \right)$$

$$= \frac{4\pi^2 f V^2 \varepsilon_0 \varepsilon_r}{\ln (R/r)} \text{ vars/mtr}$$

→ The flow of charging current causes heating of cable. Therefore load flow ~~ca~~ capability of cable is decreased.

→ Further reduction in current carrying capacity occurs due to dielectric losses.

(b) **Over voltage due to switching :-**

The interruption of capacitance gives rise to over voltage.

(c) **Ferranti effect :-**

If a significant portion of transmission and distribution system has underground cable, a rise in voltage at load point may occur under no-load condition due to Ferranti effect. Thus the voltage at load point may vary considerable from full load to no load condition.

(d) **Generator stability :-**

The ability of large generator to generator leading reactive power is limited by stability considerations. Large size generator have high synchronous reactance and their leading Mvar capability is low

HVDC Cable:-

- → there is no charging current.
- → Only I^2R losses are present.
- → Electric stress distribution in ac cable is governing by dielectric capacitance - In dc cables the stress distribution is determined by resistance of dielectric.
- → In case of ac cable the electric stress in the dielectric decreases from conductor to the sheath. In dc cable the electric stress increases and can be large at the sheath
- → dc cable have very large/high current and voltage rating. These cable have lesser insulation thickness and more compact.
- → DC cable are alway single core. The conductor is always stranded.
- → DC cable may have plastic and impregnated paper insulation. Oil filled and gas filled dc cable are also used.

POWER SYSTEM EARTHING!-

→ A number of points from generation to consumer's installations are earthed. The objects of earthing are

1. To allow sufficient current to flow safely for proper operation of protective devices.
2. To limit over voltages between neutral and ground and between line and ground.
3. To suppress dangerous potential gradients, the earth potential gradients, .

→ Earthing can be divided into neutral earthing and equipment earthing. Neutral earthing deals with the earthing of system neutral to ensure system security and protection. Equipment earthing deals with earthing of non-current carrying parts of the equipment to ensure safety of personal and protection against lighting.

(a) Earth electrode:-

A rod, pipe, plate or an array of conductors, embedded in horizontally or vertically. In distribution system the earth electrode may be a rod, about 1 m long, driven vertically into ground.

(b) Earth current!-

The current dissipated by earth by earth electrode into ground.

(c) Earth Resistance!-

The resistance offered by the earth electrode to the flow of current into ground. The resistance is not ohmic.

(d) Step potential:-

The potential difference shunted by a human body between two accessible points on ground separated by the distance of one pace assumed to be equal to one meter.

(e) Touch potential!-

The potential difference between a point on the ground and a point on an object likely to carry fault current and which can be touched by a person.

(f) Mesh potential!-

The maximum touch potential within a mesh of the grid.

(g) Transfered potential!-

A special type touch potential where a potential is transfered into or out of the substation.

TOLERABLE LIMITS OF BODY CURRENTS:-

→ The effect of electric current passing through vital organs of the body depend. on magnitude, duration and frequency of current. The most dangerous consequence is a heart condn known as ~~ventr~~ ventricular fibrillation which results in stoppage of blood circulation.

(a) Effect of magnitude of current:-

→ The threshold perception is a current of 1 mA.

→ on between range (1–6) mA are known as 'let go current', because these ~~caus~~ currents, though unpleasant, donot impair the ability of a person.

→ Current in range (9–25) mA range may be painful and impair the ability to release energised object.

→ current higher than 60 mA may lead to ~~$~~ ventricular circulation.

(b) Effect of duration of current:-

The magnitude of 50 Hz tolerable current is related to duration. According to test

50 kg weight can safely withstand the current given by equation $I_B = 0.116/\sqrt{t}$

I_B = RMS value of body current, t is the time in second.

70 kg $\quad I_B = 0.157/\sqrt{t}$ $\quad \boxed{0.03 \leq t < 3 \text{ secs}}$

(c) Effect of frequency:-

The tolerable current ~~mco~~ mentioned for (50-60) Hz.

It has been found that human body can tolerate about 5 times higher direct current.

SOIL RESISTIVITY:-

The resistivity of soil depends on the following factors:-

(a) Soil type:-

type of soil governs the resistivity to a large extent.

(b) Moisture content:-

Electrical conduction in soils is electrolytic. Soil resistivity decreases with increase in content.

(c) Temperature:-

When temp. is more than 0°c, its effect on soil resistivity is negligible. At 0°c, water in the soil starts freezing and resistivity increases.

(d) Salt content:-

The composition and amount of soluble salts also affect resistivity.

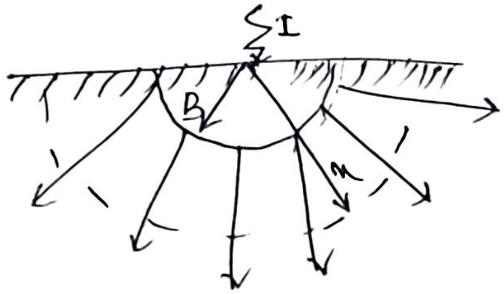
(e) Magnitude of current:-

If the value of current being dissipated by the soil is high, it may cause a significant drying of soil and increase in its resistivity.

The soil resistivity at a particular location also changes with depth. Generally, the lower layers of soil have greater moister content and lower resistivity. However if the lower layer contains hard and rocky soil, resistive may increase.

EARTH RESISTANCE:

Hemispherical Electrode:-



total resistance divided into three parts
(i) resistance of conductor
(ii) contact resistance between surface of electrode and main body of Earth.

iii) Resistance of body of earth surrounding the electrode.

First two are negligible, therefore the ~~first~~ main part of resistance is that of body of earth surrounding the electrode.

I, the current dissipated by electrode spread out radially in the earth. The current density i at a distance r from the centre of hemisphere is

$$i = \frac{I}{2\pi r^2}$$

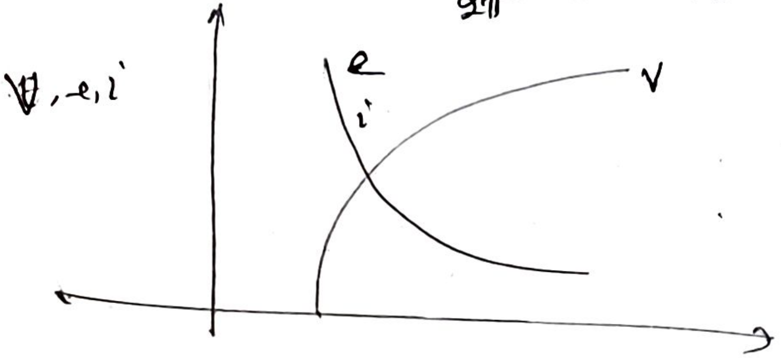
electric field due to current density i

$$\boxed{e = \rho i = \frac{\rho I}{2\pi r^2}.}$$

Voltage is the line integral of field strength e from the surface of ~~the~~ sphere of radius B to the distance r.

$$V = \int_B^r e\,dr = \int_B^r \frac{\rho I}{2\pi r^2}\,dr$$

$$= \frac{\rho I}{2\pi}\left[\frac{1}{B}-\frac{1}{r}\right]$$



The voltage between hemispherical electrode and a point at infinity (i.e $r \to \infty$) is

$$\boxed{V = \frac{\rho I}{2\pi B}}$$

The earth resistance is

$$\boxed{R = \frac{V}{I} = \frac{\rho}{2\pi B}.}$$

General Equation:-

→ Consider a system of two electrodes, Let V_1 and V_2 be the potential of these electrodes and V be the potential at any point in the medium of resistivity ρ. Let ψ be the electrostatic potential.

→ The current flow normal to surface at any point of the electrode surface is $\frac{1}{\rho}\frac{\partial V}{\partial n}$. Total flow outward from the electrod is

$$-\frac{1}{\rho}\iint \frac{\partial V}{\partial n}\, ds = -\frac{1}{\rho}\iint \frac{\partial \psi}{\partial n} \cdot ds$$

where ds is an element of electrode surface, If Q is the charge on this electrode in the analogous - electrostatic case, then by Gauss theorm

$$-\iint \frac{\partial \psi}{\partial n}\, ds = 4\pi Q$$

$$\boxed{I = \frac{4\pi Q}{\rho}}$$ total current flow

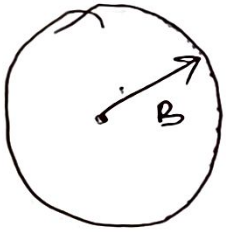
If capacity between electrodes in air in the analogous electrostatic case is C

$$\psi_1 - \psi_2 = V_1 - V_2 = Q/C$$

resistance between electrodes is R

$$R = \frac{V_1 - V_2}{I} = \frac{Q}{C}\left(\frac{\rho}{4\pi Q}\right) = \frac{\rho}{4\pi C}$$

If C is capacitance of single electrode, the return electrode being at infinity, then R is the earthing resistance of single electrode.



$R = \frac{\rho}{4\pi B}$, capacity of sphere in air is equal to radius B

If the electrode is a hemisphere buried with lower half in earth, the resistance would be doubled, gives

$$R = \frac{\rho}{2\pi B}$$

$$R = \frac{\rho}{2\pi C}$$

Driven Rod:-

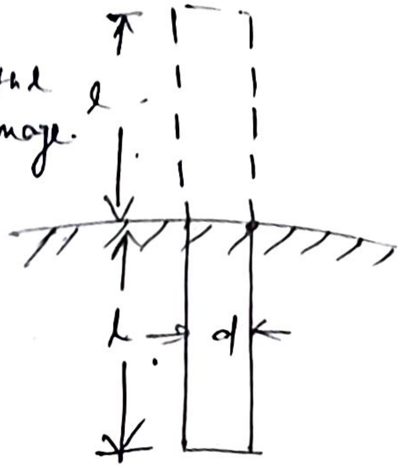


Fig show a driven rod of length l and diameter d along with its image.

capacity of rod and its image is

$$C = \frac{2l}{2\ln\frac{4L}{d}} = \frac{l}{\ln 4L/d}$$

$$R = \frac{\rho}{\frac{2\pi l}{\ln\frac{4L}{d}}} = \frac{\rho}{2\pi l}\ln\frac{4L}{d}$$

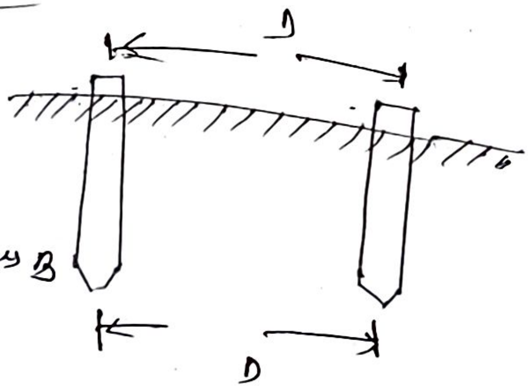
alternate expression for earth resistance of a driven rod

$$\boxed{R = \frac{\rho}{2\pi l}\left(\ln\frac{8L}{d} - 1\right)}$$

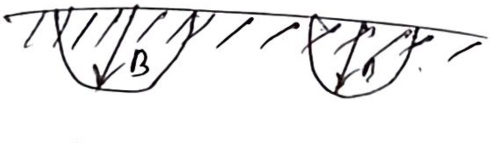
For complex ~~long~~ configuration it is usefull to replace the rod electrode by an equivalent hemisphere having the same resistance R, the radius B of hemisphere

$$\boxed{B = \frac{l}{\ln 4l/d}}$$

Multiple Rod Electrodes:-



two rod connected in parallel and at a distance D. Each rod replaced by its equivalent hemisphere of radius B carrying the charge Q.



Potential

$$V = \frac{Q}{B} + \frac{Q}{D}$$

$$= \frac{Q}{B}\left\{1 + \frac{B}{D}\right\} = \frac{Q}{B}(1+\alpha)$$

$$\alpha = B/D$$

total charge is 2Q, capacity C is equal to

$$C = \frac{2Q}{V} = \frac{2\alpha}{\frac{\alpha}{B}\cdot(1+\alpha)} = \frac{2B}{1+\alpha}$$

The earthing resistance of two parallel rod

$$\Rightarrow R = \frac{\rho}{2\pi C} = \frac{\rho}{2\pi\alpha\,\frac{2B}{1+\alpha}} = \frac{\rho}{4\pi B}(1+\alpha)$$

$$\frac{\text{Resistance of two rod in parallel}}{\text{Resistance of one rod}} = \frac{1+\alpha}{2}$$

$$\frac{\text{Resistance of three rod in parallel}}{\text{Resistance of one rod}} = \frac{2+\alpha-4\alpha^2}{6-7\alpha}$$

$$\frac{\text{Resistance of four rod in parallel}}{\text{Resistance of one rod}} = \frac{12+6\alpha-21\alpha^2}{48-40\alpha}$$

$$\frac{\text{Resistance of three parallel rods in triangle}}{\text{Resistance of one rod}} = \frac{1+2\alpha}{3}$$

$$\frac{\text{Resistance of four rods at corner of square}}{\text{Resistance of one rod}} = \frac{1+2.707\alpha}{4}$$

Earthing grid :-

The earthing system is high voltage and EHV substations consists of a number of interconnected bare conductor buried horizonally at a depth of about 0.5m. Such a system, known as earthing grid, provide common earth for all devices and metallic structures in the substations.

→ When grid depth h is less than 0.25 m

$$R = \frac{\rho}{4}\sqrt{\pi/A} + \frac{\rho}{L}$$

A → Area

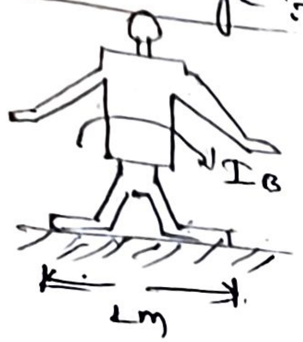
For $h > 0.25\,m$

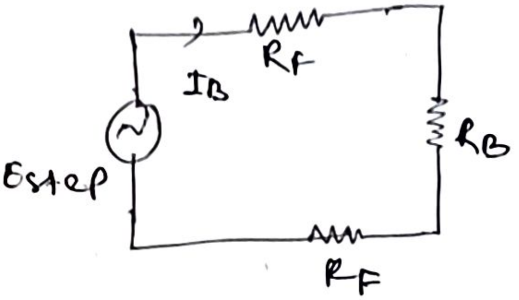
$$R = \rho \left[\frac{1}{L} + \frac{1}{\sqrt{20A}} \left(1 + \frac{1}{h\sqrt{20/A}} \right) \right]$$

TOLERABLE STEP AND TOUCH VOLTAGE :-

→ When a fault occours, the flow of current to earth results in voltage gradient on the surface of the earth. Voltage gradient may affect a person in two ways, via step on foot contact and hand to both feet on touch contact.

Step voltage :-





The potential difference shunted by the body is limited to the maximum value between two accessible points, on the ground surface, separated by one pace (assumed by equal 1 m). Rf is the earthing resistance of one foot (in ohms), and R_B is the resistance of body (in volts).

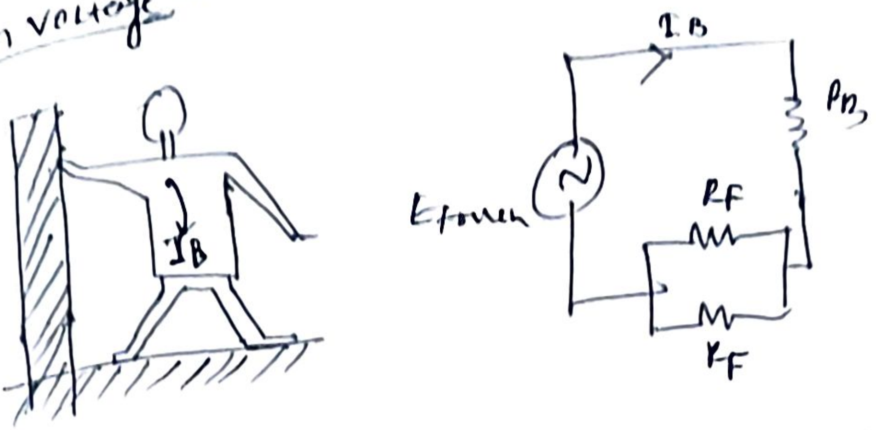
→ The tolerable value of step voltage is

$$E_{step} = (R_B + 2R_F)\, I_B \text{ Volt}$$

A human foot can be taken as equivalent to a circular plate electrode with a radius of about 0.08 m and its earth resistance may be assumed $3\rho_s$. R_B usually assumed $1000\,\Omega$

$$E_{step} = (1000 + 6\rho_s) \frac{0.116}{\sqrt{t}} \text{ volts.}$$

n voltage :-



he maximum earth potential difference intercepted would e that which would occur over a distance along the ground equal to maximum possible horizontal reach.

$$E_{touch} = (R_B + 0.5 R_F) I_B$$

$$= (1000 + 1.5 \rho_s) \frac{0.116}{\sqrt{t}} \text{ volts.}$$

transferred potential :-

A special situation occurs when a person standing within the station area touches a conductor grounded at a remote point or a person standing at a remote point touches a conductor connected to station grid.

This special case of touch potential is known as transferred potential. and can also have n value.

It is impossible to design an earthing grid for safe transfered potential, the only remedy is to isolate such conductor and to label them as live conductors.

ACTUAL TOUCH AND STEP VOLTAGES

→ Calculation of actual touch and step voltage for Small grid is very difficult and can be done only with the help of a computer.

→ Instead of calculating E_{touch} corresponding to maximum possible horizontal reach, it is preferable to calculate the maximum value of touch voltage found within a mesh of the grid. This voltage is known as mesh voltage and is given by

$$E_m = \frac{\rho K_m K_i I}{L}$$

E_m → mesh voltage, volts

ρ → Soil resistivity, ohm-m

I → Total current dissipated by grid

L = $L_c + 1.15 L_r$ for grid with rod along perimeter

= $L_c + L_r$ for grid with no rod.

L_c = total length of horizontal grid conductor

L_r = total length of ground rods

$K_i = 0.656 + 0.172 n$

n = number of parallel conductor in one direction.

$$K_m = \frac{1}{2\pi}\left[\ln\left(\frac{D^2}{16hd} + \frac{(D+2h)^2}{8Dd} - \frac{h}{4d}\right) + \frac{K_{ii}}{K_h}\ln\frac{8}{\pi(2n-1)}\right]$$

$K_{ii} = 1$ for grid with rods along perimeter or for grids with rods in grid corners as well as both along perimeter and throughout grid area.

$K_{ii} = \frac{1}{(2n)^{2/n}}$ for grid with no ground rods.

$K_h = \sqrt{1+h}$

h → depth of grid

D → spacing

d → diameter of grid conductor

Ki is known as irregularity factor and takes into account the non uniformity of current dissipation by the grid.

Step voltage $E_s = \frac{\rho K_s K_i I}{L}$

$$K_s = \frac{1}{\pi}\left[\frac{1}{2h} + \frac{1}{D+h} + \frac{1}{D}(1-0.5^{n-2})\right]$$

DESIGN OF EARTHING GRID:-

The following steps should be followed for design of an earthing grid!

(a) Data for design

- (i) Substation ground area
- (ii) Soil resistivity at the site
- (iii) Fault clearing time
- (iv) Maximum grid current
- (v) Resistivity of soil at surface

(b) Design of earthing system:-

- (i) Selection of electrode
- (ii) Determination of conductor size
- (iii) Preliminary design.
- (iv) Calculation of potential and ground potential rise
- (v)

Substation ground area and soil resistivity:-

→ Generally the substation plan is prepared before the grid design. The earth grid should cover as much ground area as possible.

→ Soil resistivity can be determined by using four probe method.

→ A thin surface layer of crushed rock helps in limiting the body current.

Fault clearing time:-

It is governed by system stability considerations and depend on protection and switchgear equipment. Generally 0.5 seconds is assumed.

Determination of maximum grid current:-

A single line to ground fault is more common and causes more fault current as compared to a double line to ground fault. Therefore the design is based on single line to ground fault current.

For calculating of this current, grid resistance and fault resistance are assumed to be zero.

$$I_g = S_f I_f$$

I_g = symmetrical grid current

I_f = rms value of symmetrical ground fault current

S_f = current division factor.

also grid current is found as

$$\boxed{I = C_f D_f I_g}$$

C_f → increase in fault current due to system growth life span of grid

D_f → Decrement factor to allow for asymmetry of the fault current wave.

Selection of Electrode Material:-

- → good conductivity
- → mechanically rugged
- → resist fusing.

→ Copper was very common used in past. It has high conductivity and is resistant to underground corrosion.

→ Aluminium is not used because of corrosion product

→ The most common is galvanised steel, the use of steel avoids galvanic action between grid and other underground structures and pipes.

- The zinc coating gets destroyed over a period of time, it is desirable that a suitable allowance for corrosion be made while determining the size of conductor.

- Generally strips are used because they can be bent, bolted and welded more easily than round bars.
- When soil is highly corrosive it is preferable to use circular rod.

Determination of Conductor size:-

$$A_c = I \left[\frac{0.00104\, \rho_m \alpha_m t}{\delta \times s \log_{10} \frac{(1+\alpha_m \theta_m)}{1+\alpha_m \theta_a}} \right]$$

A_c = crosssectional Area, mm2
I = maximum grid current, A
ρ_m = resistivity of material, μ ohm-cm
α_m = temp. coefficient of material, per °c
t = fault clearing time, second
δ = density of material, gm/cm3
s = Specific heat of material, cal/gm °c
θ_m = maximum allowable temp., °c
θ_a = ambient temp, °c